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A DIRECT READING PHASE SHIFT METER*

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ABSTRACT

A direct reading phase shift meter is described which is entirely electronic. The meter is capable of continuously indicating phase shift to within 1° in the frequency range between 10 and 1000 cps., has calibration independent of supply voltages, and is portable.

In the course of measurement of the phase shift characteristics of instruments employed in the seismic method of prospecting for petroleum, the writer felt an acute need for a phase measuring device which not only possessed the necessary accuray but which was also capable of indicating continuously the change of phase angle with frequency. Various schemes for measuring the phase shift of electrical filter networks are described in the literature. The well known oscilloscope-ellipse method, while convenient for a rapid determination of the phase angle, lacks the necessary precision, particularly when the phase angle between the two signals is in the vicinity of $\pm \pi/2$ radians. Other arrangements employing the oscilloscope as an indicating device consist of creating a voltage pulse which "rides" on the periphery of a circle produced by two voltages in quadrature.[1] This scheme is capable of indicating phase angles to within 5° of the correct value for frequencies up to 4000 cps. Still a third method involves the equalization of the magnitudes of the two vector voltages in question followed by a relatively involved process of vector addition, subtraction, and attenuation.[2] This method measures angles at discreet frequencies only

* Presented at the Annual Meeting, Houston, Texas, April 2, 1941.

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[1] Direct Reading Audio Frequency Phase Meter. W. R. MacLean and L. V. Sivian. Journal of the Acoustical Society of America *2* (April 1931), pp. 419–433.

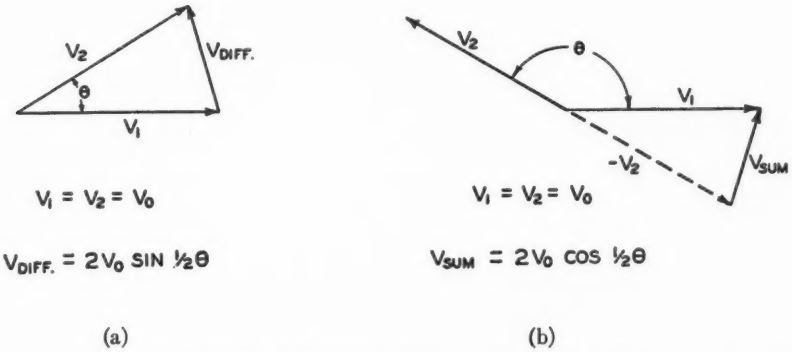
[2] Measurement of Phase Distortion. H. Nyquist and S. Brand. Bell System Technical Journal *9*, 3 (July 1930), pp. 526.

and does not as a rule give a clear picture of the shape of the phase shift characteristic until after the data have been plotted. Commercially available dynamometer type phase shift meters are designed for a single frequency only and consume enough power to render them unsuitable for work at low levels.

The meter which is to be described circumvents the limitations inherent to the devices described above. It is direct reading, is capable of measuring phase shift in the frequency range between 10 and 1000 cps., has calibration independent of the supply voltages, and is portable.

The principle of operation of this meter is as follows: If the two sinusoidal vector voltages V_1 and V_2 which are displaced from each other by a phase angle θ are subtracted, their difference by the law of cosines becomes (see Fig. 1a)

$$V_{\text{diff}} = \sqrt{V_1^2 + V_2^2 - 2V_1V_2 \cos \theta}.$$



(a) (b)

FIG. 1. Sum and difference of two equal voltage vectors.

If V_1 and V_2 are not only made equal to each other in magnitude but also to a certain predetermined magnitude V_0, then

$$V_{\text{diff}} = \sqrt{2V_0^2 - 2V_0^2 \cos \theta} = 2V_0 \sqrt{\frac{1 - \cos \theta}{2}}$$

from which

$$V_{\text{diff}} = 2V_0 \sin \tfrac{1}{2}\theta. \tag{1}$$

Similarly if V_1 and V_2 are added vectorially (see Fig. 1b) their sum will be

$$V_{\text{sum}} = 2V_0 \cos \tfrac{1}{2}\theta. \tag{2}$$

Through the proper choice of the magnitude V_0 the meter calibration is immediately established which reads directly in degrees phase. In order to prevent crowding of the meter scale Eq. (1) is used for the range of phase angles between 0 and $\pm\pi/2$ radians and Eq. (2) is used for phase angles covering the range between $-\pi/2$ to $-\pi$ or $+\pi/2$ to $+\pi$ radians. In this manner a single calibration scale may be used for the sum and the difference ranges covering all quadrants. The value V_0 is so chosen that the full scale deflection of the indicating meter corresponds to $\pm n(\pi/2)$ radians and zero deflection corresponds to 0, $\pm 2n\pi$, $\pm n\pi$ radians where n is odd. Since in reality only half angles of phase are measured the theoretical calibration scale is com-

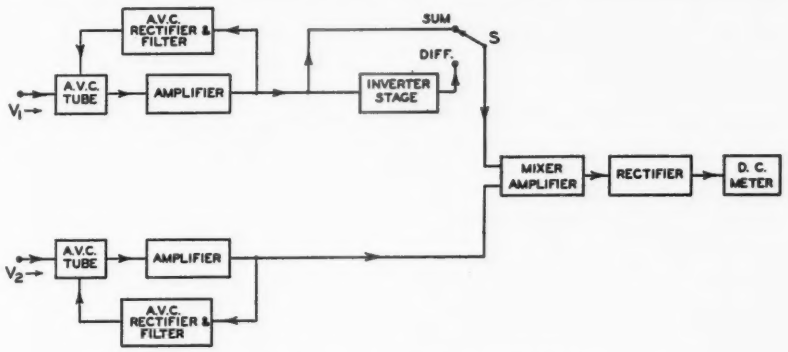


FIG. 2. Block diagram of Phase Shift Meter.

pressed at its upper end ($\pm n(\pi/2)$) by 30%. The actual calibrated scale departs from linearity by a somewhat smaller amount due to the non-linearity of the full wave rectifier ahead of the indicating D.C. meter (see below).

The principle of operation of the meter can be most easily understood by referring to the block diagram in Fig. 2. Here the two vector voltages V_1 and V_2 whose phase difference is to be determined are fed to two identical automatic volume control circuits which compensate for the variation of the magnitude of V_1 and V_2 as a function of frequency. One of the equalized vectors is then fed to a mixer amplifier stage directly and thence through a rectifier to an indicating D.C. microammeter. The other vector voltage upon equalization is fed through an inverter stage of unity gain to the mixer amplifier stage. The inverter stage is inserted only when the difference of the two vectors is read on the indicating meter and is by-passed by means of

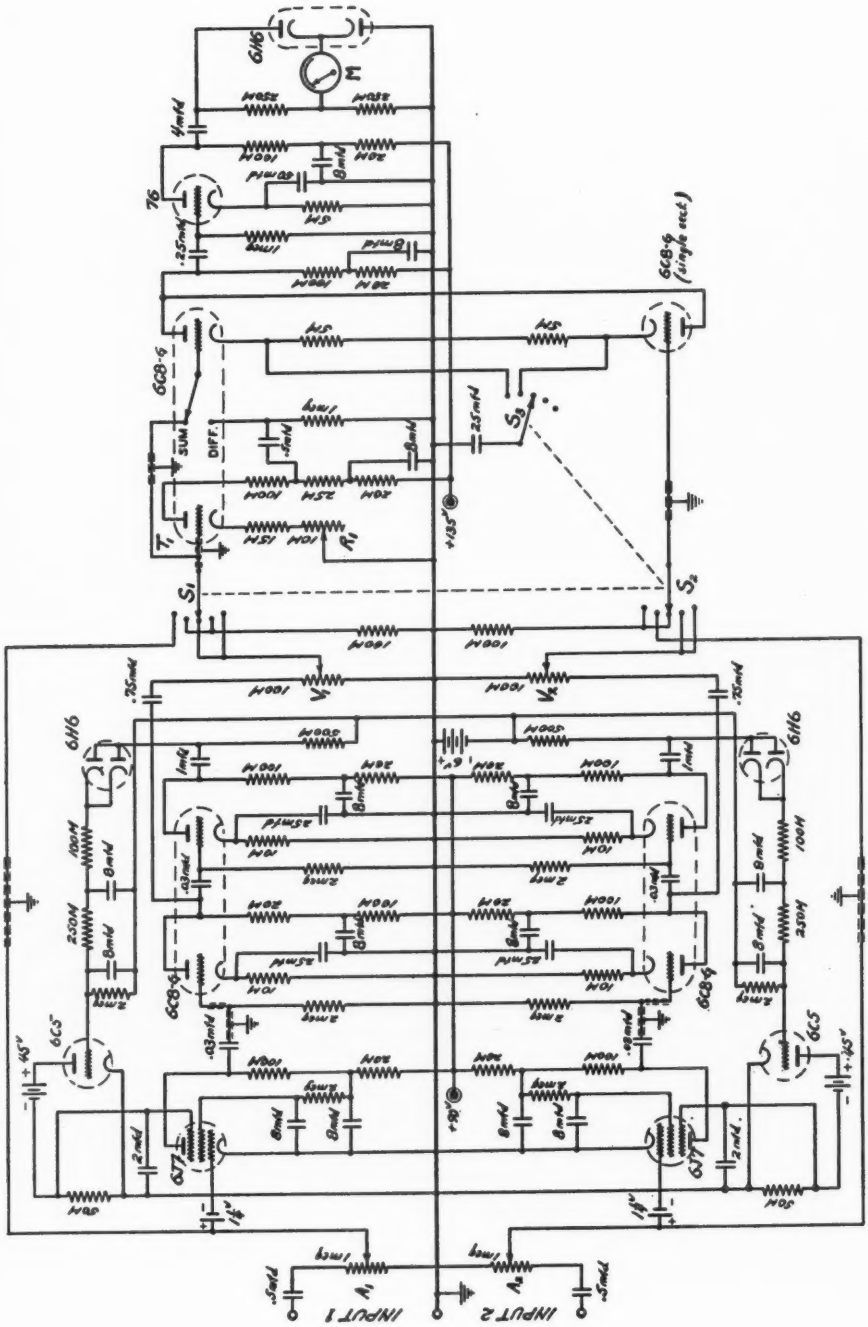


FIG. 3. Complete diagram of Phase Shift Meter.

switch S when the sum of the two vectors is read on the indicating meter. The difference or the sum position of the switch S is determined at any frequency simply by the deflection of the meter. For example, if the switch S is in the difference position (angles o to $\pm\pi/2$ radians) and the pointer moves off scale indicating a phase angle greater than $\pm\pi/2$ radians, the inverter stage is by-passed and the angle is read on the sum scale.

The complete wiring diagram of the instrument is shown in Fig. 3.

At the start of a phase measurement the input signals are attenuated manually by means of ganged selector switches S_1, S_2, S_3 and potentiometers A_1 and A_2 to a predetermined value in order that the succeeding amplifying stages will not be overdriven. The mutual conductance of each of the 6J7 pentodes is varied by changing their suppressor bias through the action of their respective feed-back loops. The latter consists of two stages of conventional resistance-capacity coupled amplifiers (6C8-G), a half wave rectifier (6H6), low pass RC filter, and a D.C. amplifier (6C5). The latter supplies additional leverage necessary for the proper performance of the automatic volume control circuit. Following the adjustment of input signals by means of potentiometers A_1 and A_2, the overall amplification of the automatic volume control loop is then adjusted by means of selector switches S_1, S_2, S_3 and potentiometers V_1 and V_2. The gain of the inverter stage T_1 is adjusted by means of an un-bypassed cathode resistor R_1. The automatic volume controls are capable of controlling a voltage change in the input signal level of 20 to 26 db to within 1% or approximately to within 1° of the correct value of the phase angle. This comparatively wide range of an accurate amplitude control necessitates a selection of 6J7 pentodes.

If a o to 50 microampere meter (M) is used as an indicating meter, the gain ahead of the mixer stages is adjusted for each individual input signal by means of potentiometers V_1 and V_2 so that $V_0=35$ microamperes. Then the reading of the phase meter for $\pm\pi/2$ will be $2\times35\times.707=49.5$ microamperes (see Eq. (1)) thus utilizing the entire working range of the microammeter. As an additional increase of the working range of the meter scale, the pointer is depressed by means of the zero adjusting screw until the emission current of the 6H6 rectifier preceding the meter is compensated for.

Since the calibration of the phase shift meter is dependent on the setting of V_0 and the latter is set manually at the start of a phase measurement, the permanence of calibration is affected only by the change of the characteristics of the 6H6 rectifier ahead of the indicating

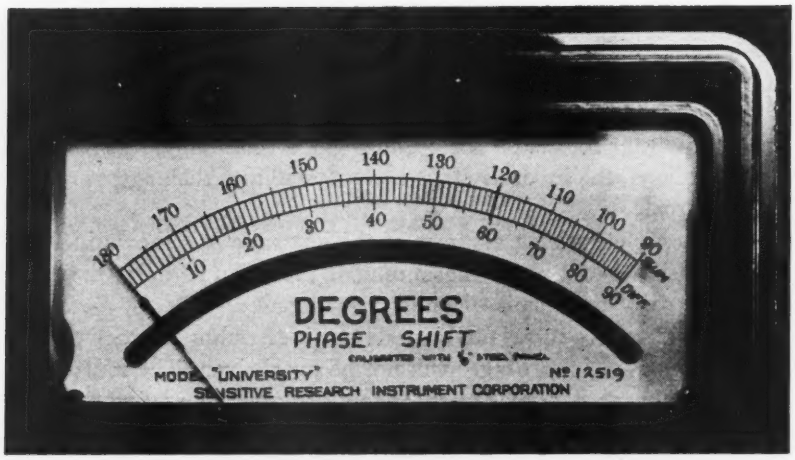


FIG. 4. Scale of Phase Shift Meter.

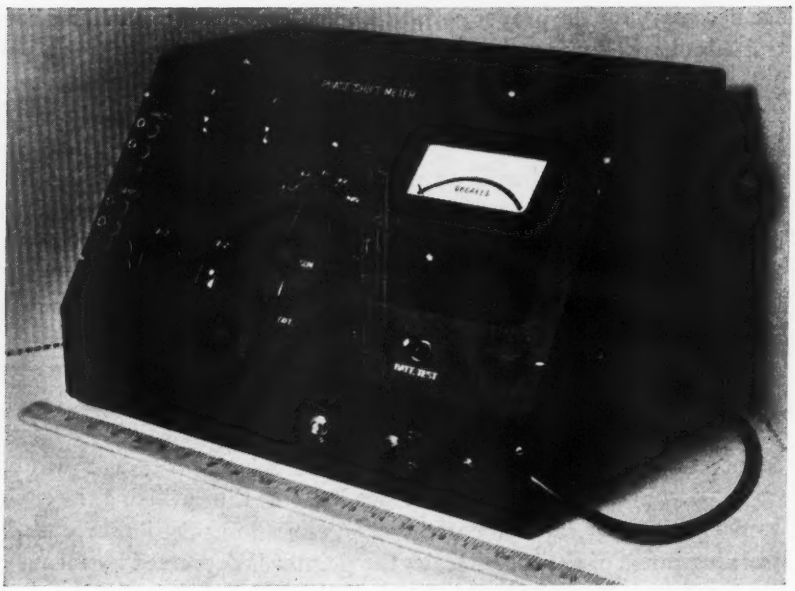


FIG. 5. Phase Shift Meter, complete.

meter and the meter itself. The calibration of the meter is carried out by means of a simple RC network wherein the phase angle is computed

as a function of frequency. A photograph of an actual calibrated scale is presented in Fig. 4 showing the proper coincidence of the sum and the difference scales.

In the arrangement described above the voltages are supplied entirely by means of "A" and "B" batteries in order to minimize 60 cps. pickup in the apparatus under measurement. The external view of the phase shift meter is shown in Fig. 5.

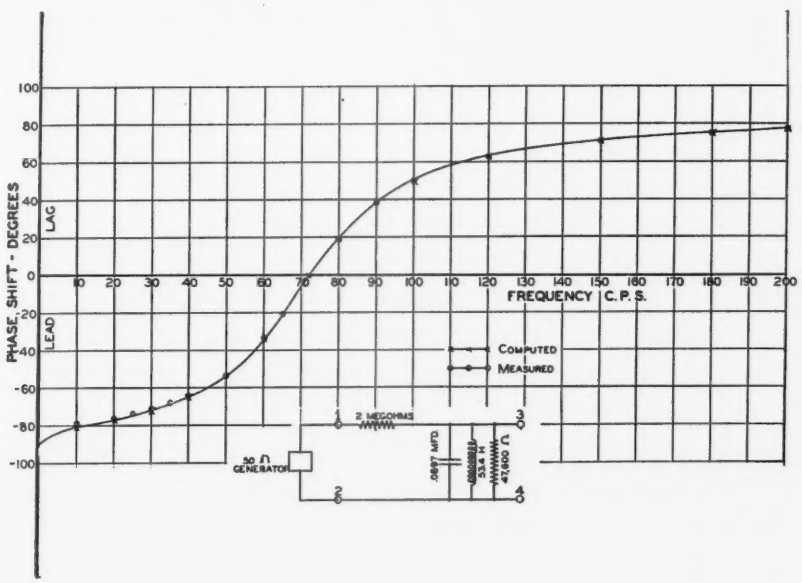


FIG. 6. Performance of meter showing measured and computed phase shift with frequency of a simple damped LC circuit.

The performance of the meter described above is illustrated in Fig. 6 wherein a comparison is made between the measured and the computed phase shift characteristic (between points 12 and 34) of a simple damped anti-resonant LC circuit. In computing the phase characteristic of this circuit the variation of Q of the choke with frequency was taken into account.

In conclusion the writer wishes to express his thanks to Dr. J. A. Sharpe for his numerous valuable suggestions and to Stanolind Oil and Gas Company for their kind permission to publish this paper.

THE RELATION BETWEEN DEPTH, LITHOLOGY, AND SEISMIC WAVE VELOCITY IN TERTIARY SANDSTONES AND SHALES*

N. A. HASKELL†

ABSTRACT

Velocity data in Tertiary sands and shales, obtained at 62 wells in the southern San Joaquin Valley, California, are considered statistically. A mean vertical velocity gradient of 0.464 feet per second per foot is found, and evidence is presented which indicates that, on the average, 0.360 feet per second per foot, or 78% of the total vertical gradient, is due to the effect of the pressure exerted by the present over-burden. It is found that in shales the effect of overburden is smaller than this average (0.240 and 0.231 feet per second per foot for two shale formations), and in sandier formations it is greater than the average (0.428 and 0.581 feet per second per foot for two moderately sandy formations). However, the velocity vs. depth curves for sands and for shales may intersect at any depth, and it does not appear to be possible, in general, to distinguish between the two solely on the basis of the velocity at a given depth.

Comparison with similar data of Weatherby and Faust for older beds and with the experimental results of Birch and Bancroft for hard sandstone and slate suggests that these gradients cannot persist very far beyond the present maximum depth of observation of about 14,000 feet.

INTRODUCTION

There are two related questions to which an answer is desired in order to make the fullest possible use of the data derived from well velocity determinations. These are:—

(1) How much of the average increase of velocity with depth is due to the purely mechanical effect of the pressure exerted by the present overburden, and how much is to be attributed to the intrinsic character of the various beds themselves?

(2) To what extent can the velocity log of a well be related to the lithologic character of the strata traversed by the well?

An answer to the first question would provide a basis for estimating the effect of structural relief on the velocity distribution. The second question, while not of great practical importance at present, might become significant if methods are developed to measure velocities accurately over small intervals. Both are particularly pertinent in areas of poorly to moderately compacted sediments, such as the California Tertiary basins and the Gulf Coast.

* Presented at the Annual Meeting, Houston, Texas, April 2, 1941.

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The best experimental work[1] on the effect of pressure on velocity relates to the velocity of transverse waves in extremely rigid, high velocity rocks, principally igneous and metamorphic, and is not directly applicable to the problem under discussion. The principal evidence must therefore come from well velocity determinations.

The present study is based on velocity surveys obtained at 62 wells in the southern San Joaquin Valley, California, most of which have been conducted under the auspices of the Cooperative Well Velocity Surveying Group. Of these wells 44 are located in Kern County, 13 in Fresno County, 2 in Tulare County, and one each in Kings, Madera, and Merced Counties.

Methods

The most direct attack on the problem of the effect of overburden would be to select some definite stratigraphic unit and determine the velocity through the selected bed at a number of wells scattered over an area in which the depth of the bed varies over a wide range. There are, however, several objections to this method of approach. In most well velocity determinations the positions at which measurements are made bear no relation to stratigraphic boundaries, and the interval velocities obtained do not, therefore, necessarily correspond to the velocities of any definite lithologic units. Furthermore, unless one can average a number of determinations from different wells at approximately the same depth, the individual determinations may be so erratic, due to observational errors, that no reliable conclusions can be drawn, especially if a thin stratum is used. Finally, and most important, if the results are to have any significance, the chosen stratum must be composed of the same type of rock at all wells used, that is, there must be no lateral variations in lithology along the bed. This is particularly unlikely to be the case in an area of the type from which the present data are drawn, where rapid lateral variations in lithology are the rule rather than the exception.

An alternative method is to select a stratigraphic zone or group whose thickness is considerably greater than the seismometer interval used in determining the velocities at the wells, and average all interval velocity determinations within this zone which are obtained over approximately the same depth range. Since the average for each depth range will then be derived from a considerable number of wells scat-

[1] F. Birch and D. Bancroft. The effect of pressure on the rigidity of rocks. Jour. Geol. 46, 59 and 113 (1938) and Jour. Geol. 48, 752 (1940).

tered over a wide area, there will be a tendency to average out both the observational errors and the effects of lateral lithologic variation. On the other hand, if the thickness of the chosen zone is comparable to its total relief from the wells in which it is shallowest to those in which it is deepest, the greater part of the data for the shallower depths will be derived from the upper part of the zone and that for the greater depths will be derived from the lower part. Thus, in addition to the effect of increasing overburden, there may be an additional change in velocity due to any lithologic changes within the zone from top to bottom, which is just the effect we wish to eliminate in order to arrive at the component due to overburden alone, If a stratum which is much thinner than its maximum relief is used, the average for any given depth will be taken over the whole thickness of the stratum and the effect of lithologic variation from top to bottom will be eliminated. This limiting case, however, is exactly equivalent to the method discussed in the preceding paragraph and is open to the same objections. Nevertheless, if we consider a number of different zones of comparable lithologic character, but of progressively diminishing thickness, the corresponding values found for the apparent rate of increase of velocity with depth should, ideally, show a progressive trend toward the limiting "thin layer" value, which gives the effect of overburden alone. If such a trend is actually found, we are justified in having a certain amount of confidence that the various extraneous factors, which might affect the validity of the result, have in fact been averaged out.

Results

In spite of the great local variability of the lithology of the various members of the stratigraphic section of the San Joaquin Valley, the major divisions do exhibit certain characteristic features when considered as a whole. Thus the Upper Miocene and the Oligocene (Kreyenhagen) are dominantly shales,* the Middle and Lower Miocene and the Eocene contain a relatively large proportion of sandstones, while the average character of the Pliocene is probably intermediate between that of the other two groups. In applying the method described above to each of these five zones, the interval velocities over

* At some of the Kern County wells the Upper Miocene contains the thick Stevens sand zone. Of these, many were drilled to, but not deeply into the sand, so that their interval velocities do not show the effect of this zone. Those which do penetrate the sand are not sufficiently numerous to counterbalance the generally shaly character of these beds over most of the area.

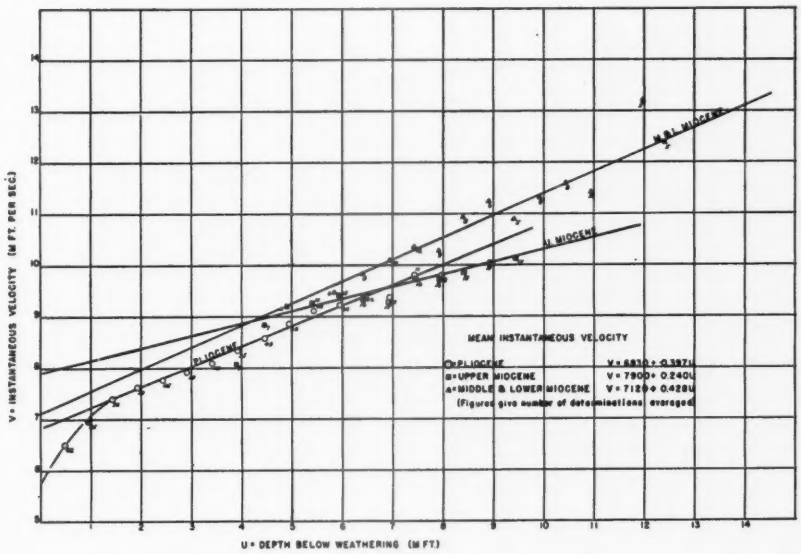
V = INSTANTANEOUS VELOCITY (M FT. PER SEC.)
U = DEPTH BELOW WEATHERING (M FT.)
M.&L. MIOCENE
U. MIOCENE
PLIOCENE
MEAN INSTANTANEOUS VELOCITY
P=PLIOCENE V = 6830 + 0.397U
a=UPPER MIOCENE V = 7900 + 0.240U
a=MIDDLE & LOWER MIOCENE V = 7120 + 0.428U
(Figures give number of determinations averaged

FIG. 1.

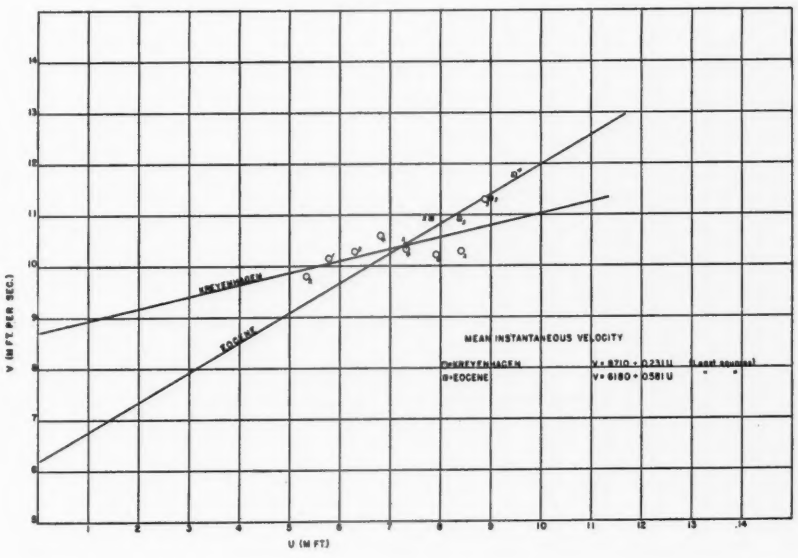
V (M FT. PER SEC.)
U (M FT.)
KREYENHAGEN
EOCENE
MEAN INSTANTANEOUS VELOCITY
K=KREYENHAGEN V = 8710 + 0.231U (least squares)
E=EOCENE V = 6180 + 0581 U

FIG. 2.

1000 foot intervals, determined from the well survey data, were tabulated with the depth below weathering of the mid-point of the interval. All velocities and mid-point depths for intervals over approximately the same depth range in the same zone were then averaged. These mean points are plotted in Figs. 1 and 2.

No doubt, if a sufficient number of observations at very shallow depths were available, all these formations would show an initial rapid increase of velocity followed by a slower, approximately linear increase, as is the case with the curve for the Pliocene. Although, since some poorly consolidated Pleistocene, or even Recent, beds may be included with the Pliocene in the shallowest points, the initial rate of increase may be abnormally great in this case. The intercepts of the straight lines drawn through the plotted points do not, therefore, necessarily represent the velocities which would actually be measured in these beds at the surface.

It is noteworthy that the sandier formations (Middle and Lower Miocene and Eocene) show conspicuously greater rates of increase of velocity with depth (0.428 and 0.581 feet per second per foot respectively) than the shales (Upper Miocene, 0.240, and Kreyenhagen, 0.231). However, since the velocity vs. depth curves for a typical sandstone and a typical shale may intersect at almost any depth, the problem of deducing the lithologic character of a given bed solely on the basis of its velocity at a given depth appears to be essentially indeterminate. The value of dv/du, rather than the velocity itself, seems to be more directly related to the lithology. Porosity is presumably the principal factor involved.

As pointed out above, these figures for dv/du do not necessarily represent the component due purely to the effect of increasing overburden, since the zones used average considerably more than 1000 feet in thickness. Fig. 3 illustrates the type of data which one obtains by going to the "thin-layers" extreme. These points were derived by estimating the instantaneous velocity at a number of wells in the beds immediately above a particular horizon, in this case the Vedder sand. It will be noted that, in spite of the wide scattering of the individual points, the steepest gradient that can reasonably be derived from these data is comparable to those found above for the sandier formations, and the flattest gradient is comparable to those for the shales, while the mean is nearly equal to the average of the zones previously considered. Similar estimates have been made of the instantaneous velocity immediately above three other well established marker beds in

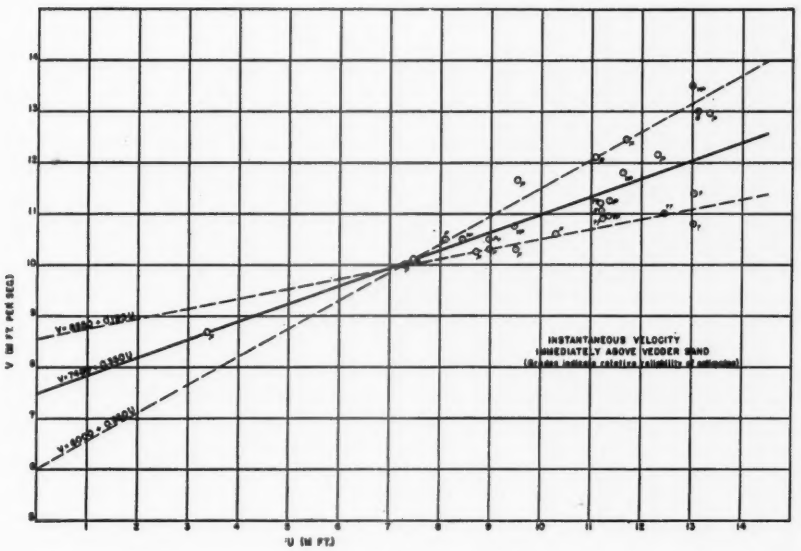
INSTANTANEOUS VELOCITY
IMMEDIATELY ABOVE VEDDER SAND
V (M FT PER SEC)
U (M FT)

FIG. 3.

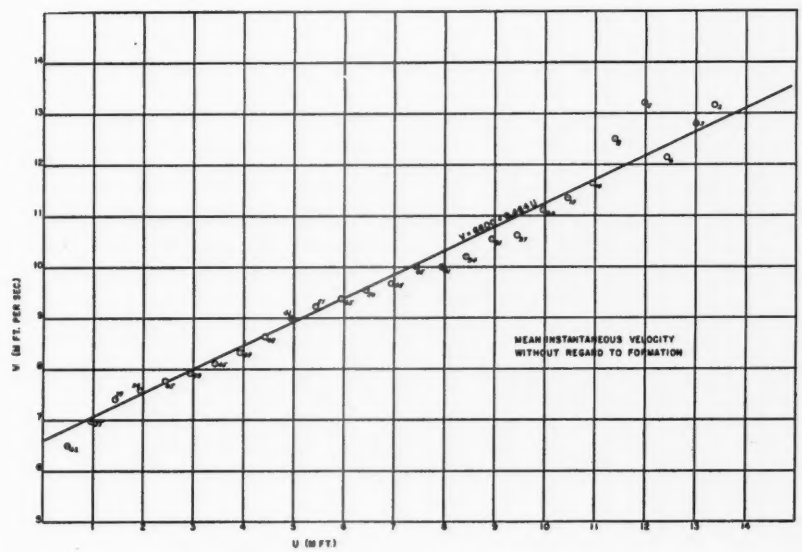
MEAN INSTANTANEOUS VELOCITY
WITHOUT REGARD TO FORMATION
V (M FT. PER SEC)
U (M FT)

FIG. 4.

the San Joaquin Valley. The mean gradients were found to be 0.448, 0.246, and 0.396 feet per second per foot. The average of the four "thin-layer" gradients is 0.360 feet per second per foot.

If we go to the opposite "thick-layer" extreme by averaging all velocity determinations over the same depth range without regard to formation, we find the values plotted in Fig. 4. The mean gradient of 0.464 feet per second per foot in this case represents the effect of overburden plus the full effect of variations in lithology with stratigraphic depth. As the next step in the elimination of the latter effect we may take the gradient for the Pliocene alone, and, as a further step, the mean of the values found for the pre-Pliocene formations, since each of these individually has a mean thickness considerably less than that of the Pliocene and the average of all is probably more nearly comparable lithologically to the Pliocene, and to the section as a whole, than any one of them alone. We then have the sequence of values of dv/du shown in Table I. These figures exhibit a consistent trend in the direction that was to be expected from the method of averaging used. Therefore the lower limit is probably a fairly good estimate of the mean rate of increase of velocity due to overburden alone, for beds having the character of the average of the Tertiary section as a whole. It thus appears that 0.360/0.464, or approximately 78%, of the overall rate of increase of velocity with depth in this area is due to overburden pressure, the remainder being due to the older formations being, on the average, more firmly cemented.

TABLE I

MEAN VALUES OF dv/du FOR PROGRESSIVELY THINNER ZONES

Zone	*Mean dv/du*
Whole Tertiary Section	0.464 ft/sec/ft
Pliocene	0.397
Mean of 4 pre-Pliocene Groups	0.370
Mean of 4 Thin Strata	0.360

Comparison with Other Data

A similar statistical treatment of well velocity data from the mid-Continent and Gulf Coast areas was carried out a number of years ago by Weatherby and Faust[2] in order to determine the difference in velocity in beds of comparable character varying in age from Devonian to Tertiary. They were not primarily concerned with isolating the

[2] B. B. Weatherby and L. Y. Faust. Bull. A.A.P.G. 19, 1 (1935).

component of the gradient due to overburden alone, nor in making a distinction between the behavior of sands and shales. Moreover, their data did not extend below 5000 feet. However, a comparison between their results for the Tertiary and the gross average without regard to formation of the present discussion may be of some interest. This is shown in Fig. 5. The steeper slope indicated by their data does not necessarily imply a much greater gradient due to overburden in these

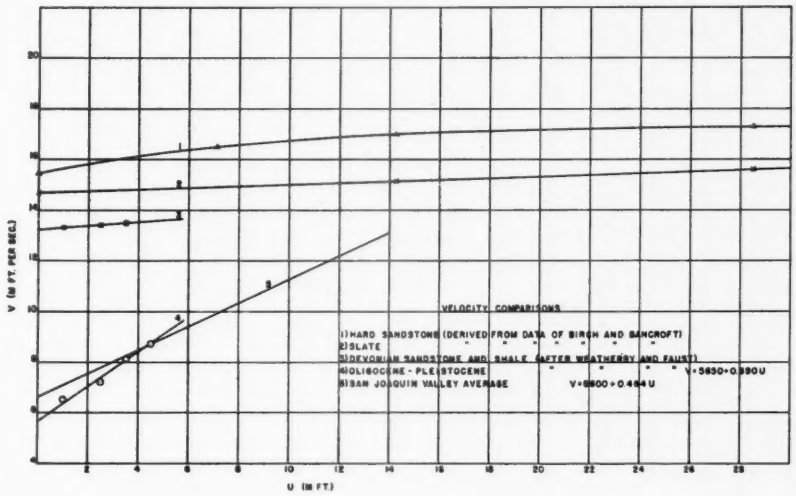
V (IN FT. PER SEC.)
U (IN FT.)
VELOCITY COMPARISONS
(1) HARD SANDSTONE (DERIVED FROM DATA OF BIRCH AND BANCROFT)
(2) SLATE
(3) DEVONIAN SANDSTONE AND SHALE (AFTER WEATHERBY AND FAUST)
(4) OLIGOCENE - PLEISTOCENE V=5830+0.390U
(5) SAN JOAQUIN VALLEY AVERAGE V=6600+0.464U

FIG. 5.

beds. Of the total mean gradient of 0.464 for the San Joaquin Valley, the portion due to overburden has been estimated as 0.360, leaving 0.104 for what we may call the "stratigraphic component" of the gradient. Both the California data and that of Weatherby and Faust cover approximately the same range of geologic time. However, in the former the stratigraphic effect is distributed through a maximum depth range of about 13,500 feet, while in the latter the corresponding depth range is 4500 feet. If we assume the total stratigraphic effect to be the same in the two cases, the stratigraphic component of the gradient is 13.5/4.5 times as great for the thinner section. Thus out of the total gradient of 0.690, the stratigraphic component is 0.312, leaving 0.378 for the portion due to overburden. This is not appreciably different from that which we have found from the California data.

Other curves shown in Fig. 5 are Weatherby and Faust's average

for Devonian sandstones and shales, and two curves derived from the measurements of Birch and Bancroft[3] on specimens of a slate and a sandstone (presumably quartzitic). These measurements give the transverse wave velocity, V_s, as a function of pressure. For comparison with the present data the pressures have been converted into the equivalent depths in a sedimentary section of mean density 2.3, and the longitudinal wave velocity, V_p, has been computed on the assumption of a constant ratio, $V_p/V_s = 1.55$. Since we are interested only in comparing the well velocity data and the experimental results in order of magnitude, this assumption of a constant ratio is sufficiently accurate.

It seems unlikely that pressure compaction can do more than change the soft Tertiary sediments into materials resembling the slate and sandstone used in these experiments. If this is the case, the roughly linear increase of velocity with depth, obtained from the well data, cannot be extrapolated very far beyond the present maximum depth of observation of about 14,000 feet. Below this depth the gradient probably begins to diminish toward an asymptotic value in the neighborhood of 0.02 to 0.03 feet per second per foot.

[3] Loc. cit. 3rd paper.

SEISMIC VELOCITIES IN THE SOUTHEASTERN SAN JOAQUIN VALLEY OF CALIFORNIA*

E. J. STULKEN†

ABSTRACT

For the first time, seismic velocity measurements from well surveys have been made intensively enough to justify an analysis of the velocity field in an entire area instead of just along lines between wells. Maps are drawn showing velocity changes in the southeastern San Joaquin Valley of California. A portion of the valley floor in the neighborhood of Bakersfield, about twenty-five miles wide and thirty-five miles long, was chosen for study because of the number of wells in the area whose velocities were known. Differences in average velocity of 1700 feet per second for a constant depth are observed, and horizontal velocity gradients averaging over 100 feet per second per mile are computed. Correction schemes for the adjustment of seismic data are suggested, and correction maps shown.

An attempt is made to establish a connection between stratigraphy and seismic velocity. Comparative study of the logs of wells and the velocities observed in them yields certain qualitative conclusions, but attempts to express the relation in a quantitative way fail.

INTRODUCTION

Computation of seismic data has in general been carried out with the aid of a simple time-depth relationship, in which velocity varies with depth alone. Such relationships are usually based on velocity profiles or seismic well-survey information, and for application over a whole area any irregularities and discontinuities which they contain are smoothed out until a fairly regular curve or succession of values is obtained. In areas where few wells or velocity profiles have been shot, such simplified functions are the best available; with their help invaluable information has been uncovered, in spite of the fact that the number and prominence of the reflections often show that there must be many velocity discontinuities in the section.

However, in certain areas, discrepancies have been observed in commercial work and in neighboring velocity measurements that were serious enough to necessitate the closer consideration of the problem of seismic velocity. Sometimes even the geology and physical geography of the region suggest strongly that there may be important lateral velocity variations. These conditions apply with especial force to that part of Kern County, California, which is in the San Joaquin Valley, both because of the type of sedimentation and because the area has been so thoroughly explored and drilled.

The San Joaquin Valley, a part of the Great Valley of California, is a conspicuous trough blocked off at the southeast end by the Te-

* Presented at the Annual Meeting, Houston, Texas, April 2, 1941.

† Allen Academy, Bryan, Texas.

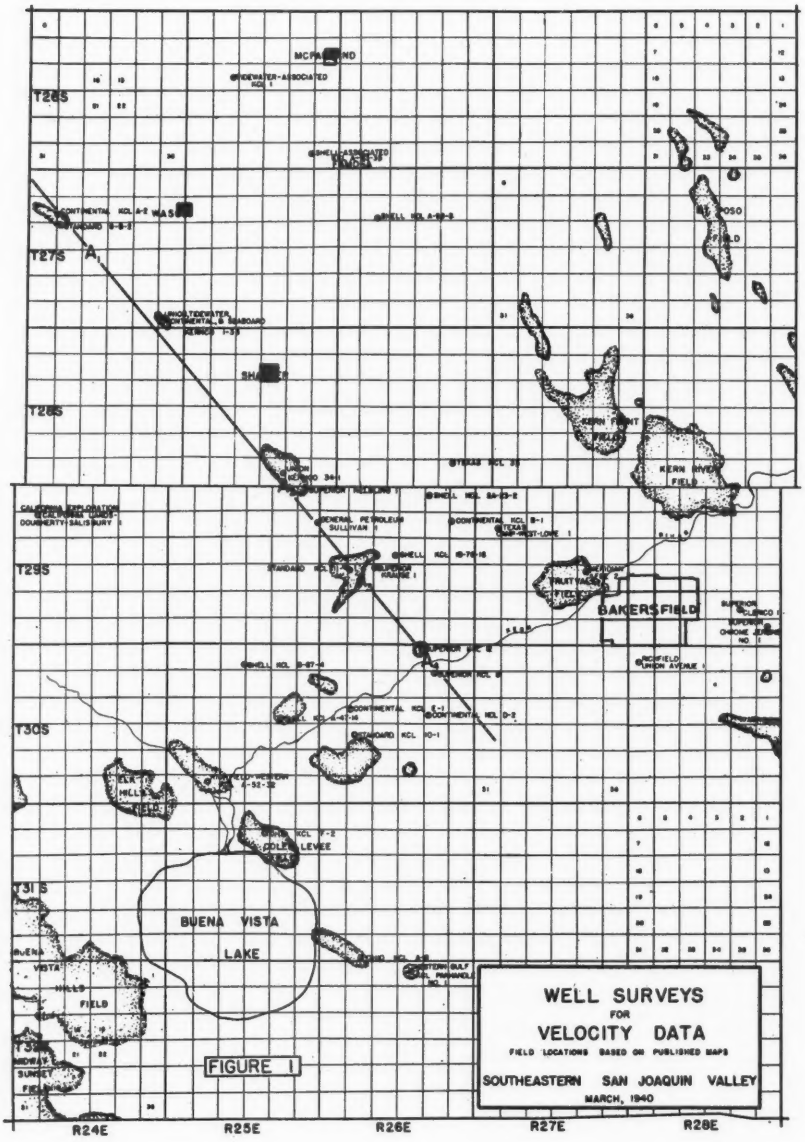

FIG. 1. Location of well velocity surveys in S. E. San Joaquin Valley, California.

hachapi and San Emigdio mountains, and running in a northwesterly direction between two great mountain chains, the Sierra Nevada on the northeast and the Coast Ranges on the southwest. In spite of the fact that the valley floor forms in general a plain, its geology is far from simple. Faults are present, unconformities numerous, and thickening or lensing out of beds is evident in many places. Furthermore, the character of given horizons changes so from well to well that a comparison of well logs often indicates greatly different lithologic conditions.

The need for extensive velocity data resulted in the formation of the Cooperative Well Velocity Surveying Association, which has, since June, 1938, made velocities available from over 35 wells in the area to be studied. This collection of surveys has demonstrated the presence of important lateral variations in velocity, and has given us as complete a set of quantitative velocity information as can be found anywhere. The locations of these wells are given in Fig. 1, which shows their relation to towns and oil fields, as well as to townships and sections. Although wells in other areas have been surveyed by the Association, so that maps could have been made over a larger area, the present study is restricted to the region where the density of surveyed wells is high and the stratigraphic correlations relatively reliable.

Average Velocities at Constant Depth

The first two contour maps (Figs. 2 and 3) show contours representing the average velocity, measured from the bottom of the weathered layer down to 5000 and 7500 feet. The velocity at any point is greater at 7500 feet than at 5000, and the contour patterns on both maps are similar enough so the area can be divided roughly into zones of low, medium, and high velocity. The most interesting thing about the maps is the unexpectedly large changes in velocity. If the velocity were a function of depth alone, as is usually assumed, and did not change laterally, there would be no contours on the maps; the number of contours illustrates the importance of the lateral changes. The horizontal gradient, or rate of change of the average velocity per mile, is measured by the spacing of the contours; close spacing of course means a large gradient. The maps show that the horozintal gradient is of the order of one hundred feet per second per mile.

The size of the velocity changes is illustrated in another way by Fig. 4, which is a graph of the average velocity versus the horizontal

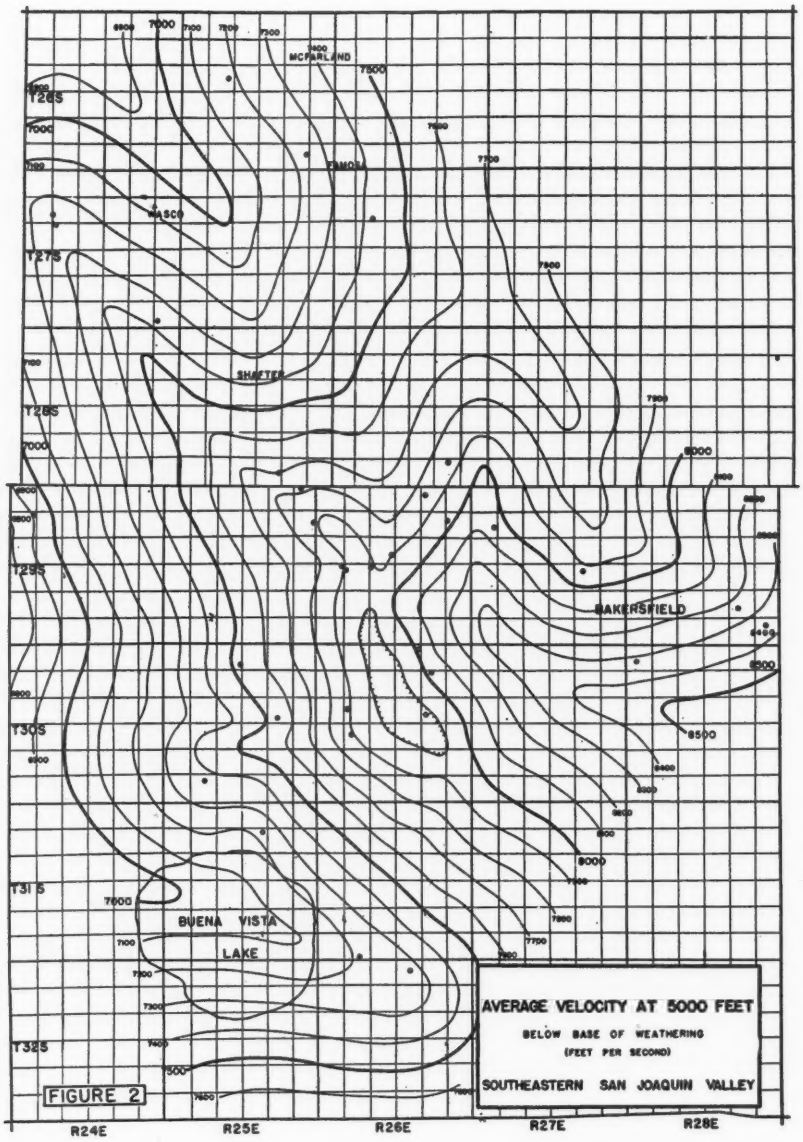

FIG. 2. Average seismic velocity at 5000 ft. from well surveys.

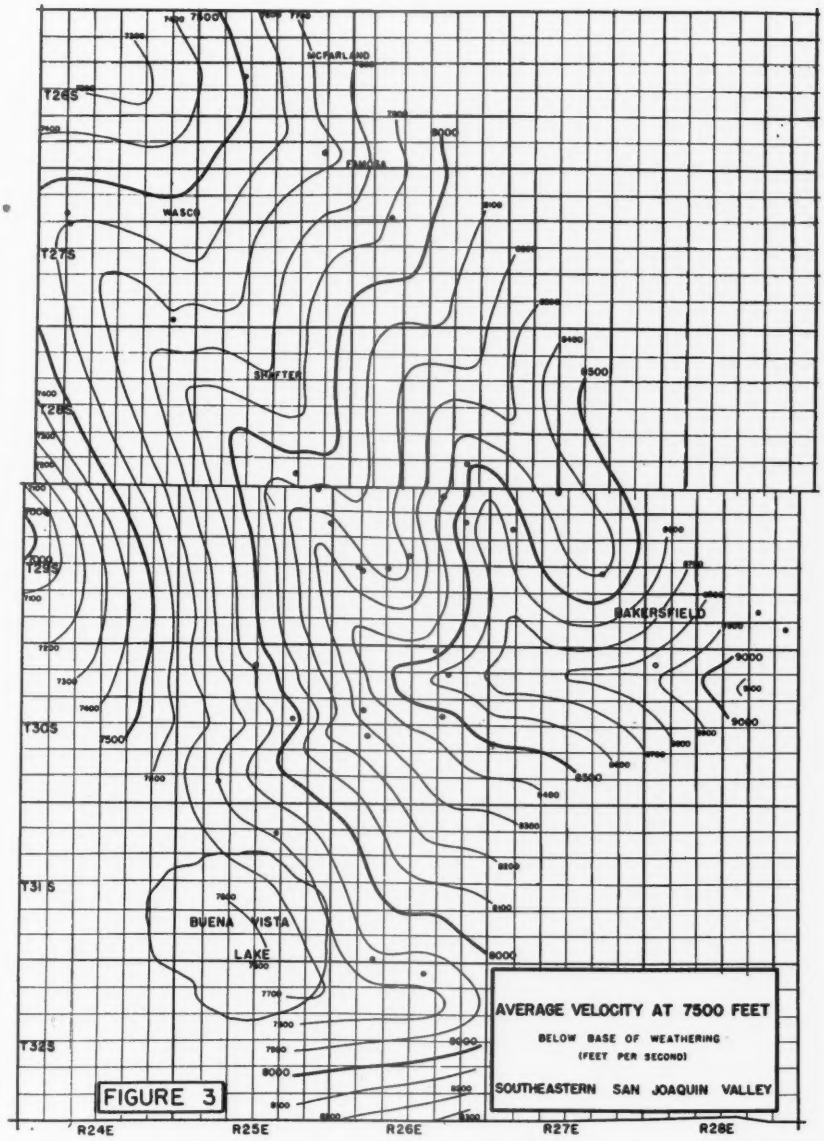

FIG. 3. Average seismic velocity at 7500 ft. from well surveys.

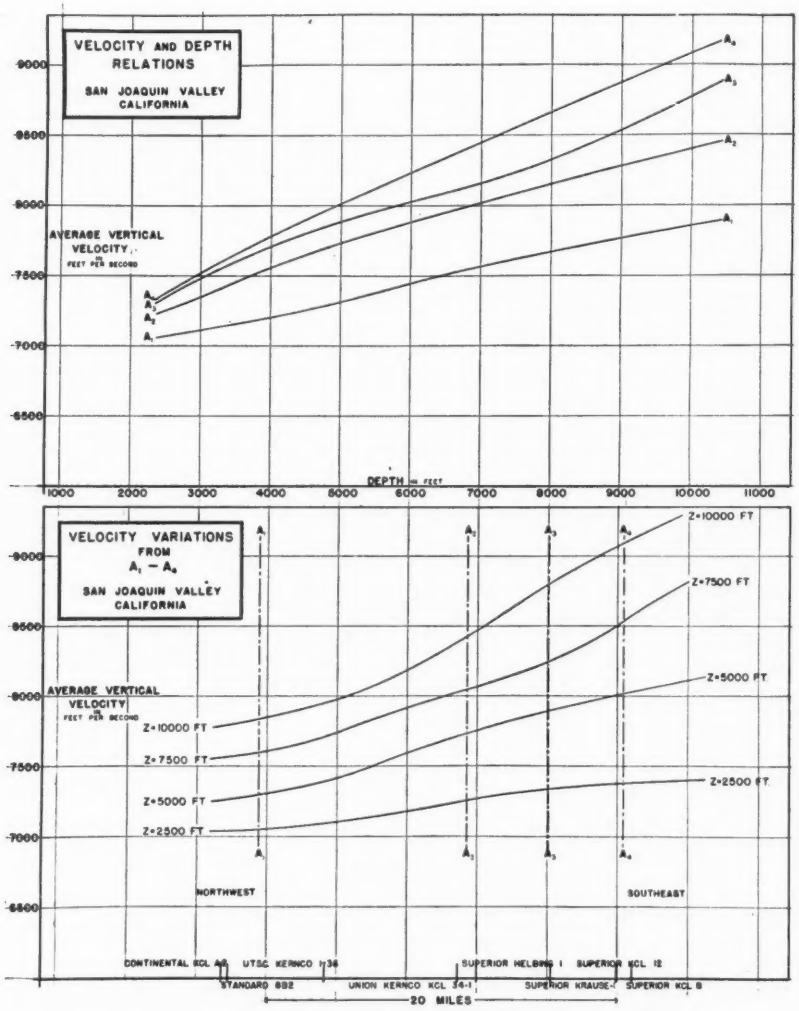

FIG. 4. Velocity vs. depth relations from four locations along line A_1-A_4 of Fig. 1. Velocity variations vs. location along the above line.

location for four different depths. The locations are along the line A_1-A_4, as shown in Fig. 1. The graph shows that the average velocity to ten thousand feet, at one end of the line, is less than that to five thousand feet at the other end. The analytical consequence of these data is that the horizontal gradient along the line is about one-twentieth that of the average vertical gradient, or that the "surfaces of constant velocity" have an average slope of five per cent.

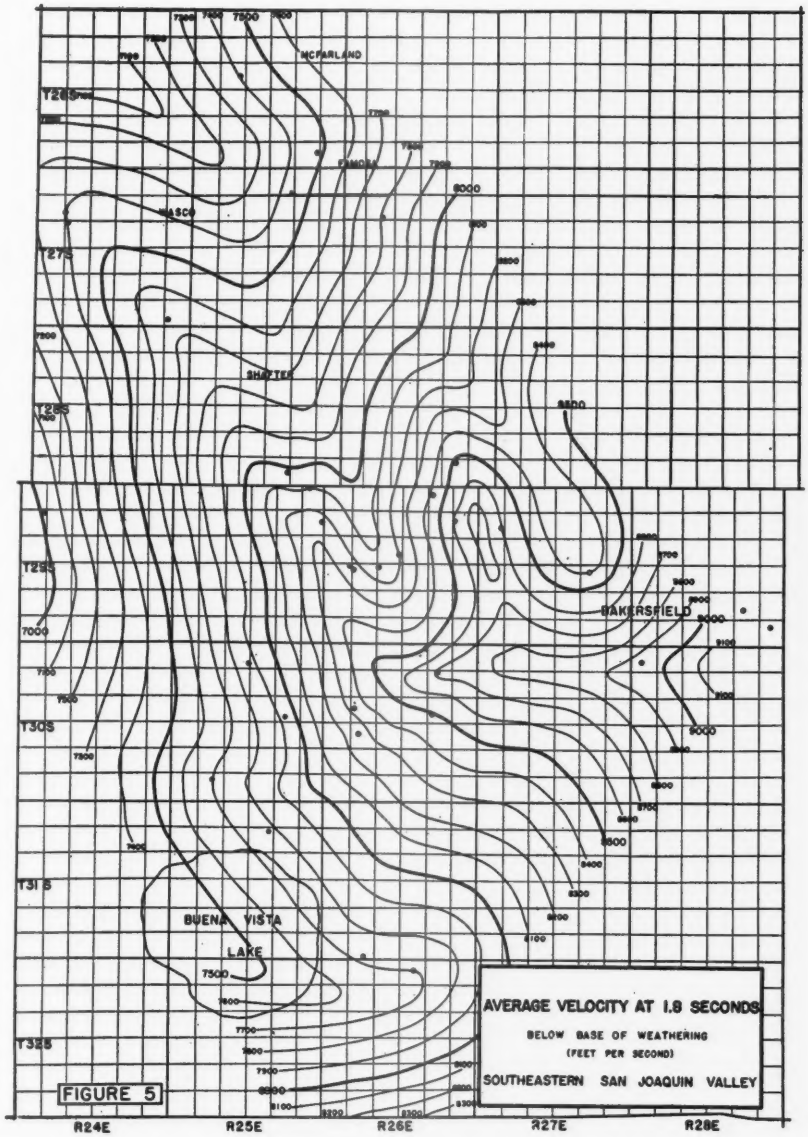

FIG. 5. Contours of average velocity at constant arrival time of 1.8 seconds.

Average Velocities at Constant Arrival Time

Figs. 5 and 6 show contours of the average velocity at constant arrival times of 1.8 and 2.2 seconds. These contours show that the veloci-

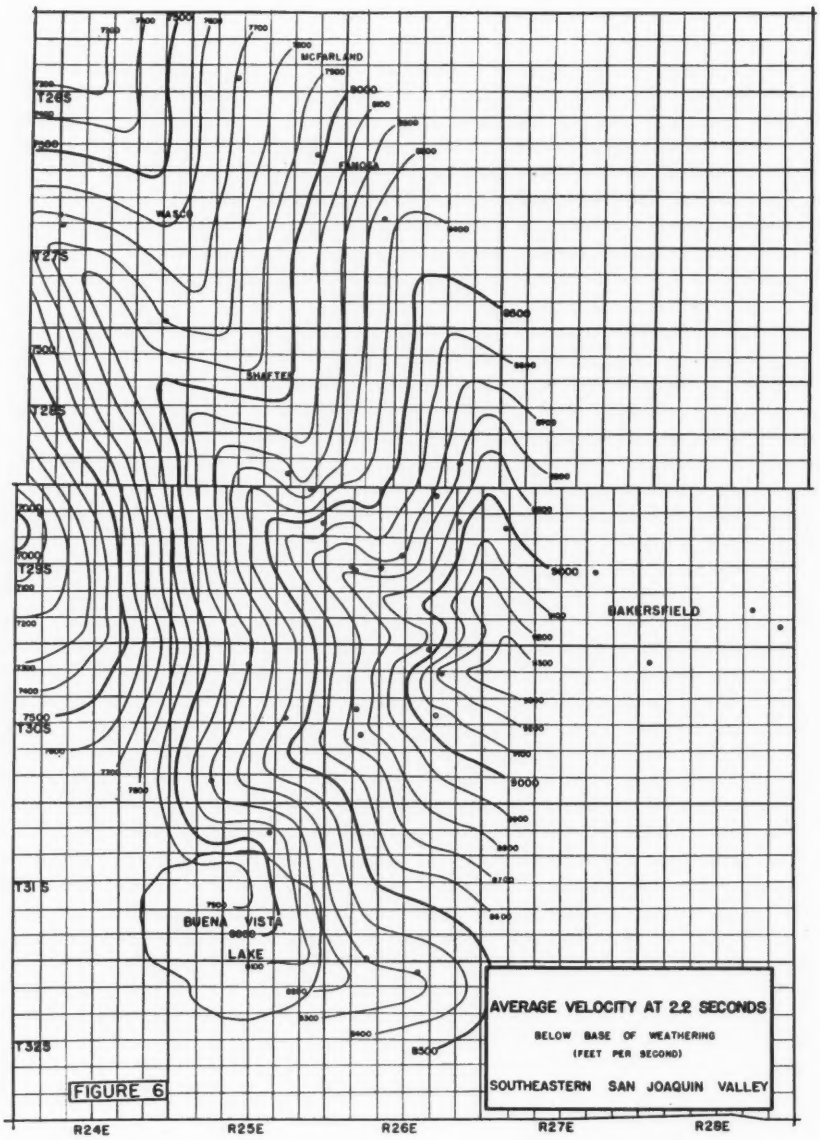

FIG. 6. Contours of average velocity at constant arrival time of 2.2 seconds.

ties differ by more than two thousand feet per second from one side of the area to the other. Time-depth charts used for routine depth computations generally allow the velocity to depend only on the arrival time. Figs. 5 and 6 show that when such a chart is used in the area under consideration, a seismic horizon whose depth in the middle of the area is correctly computed will be mapped a thousand feet too shallow on one side and a thousand feet too deep on the other. The seriousness of the matter is apparent when it is remembered that the whole area is smaller than an average county. Another way to look at it is to point out that a seismograph map computed with a single time-depth relation would contain a cumulative depth error of over a hundred feet per mile. This is enough distortion to make a typical major structure look like a shelf.

CORRECTIONS FOR SEISMIC COMPUTATIONS

Simple Corrections

The simplest way to apply well velocity data is to use individual well surveys as time-depth charts for the immediate neighborhood of the well. However, this leads to trouble when seismic data between two surveyed wells are considered, for at intermediate points such data will not necessarily correspond to conditions at either well. For this reason, a commonly used method is to interpolate between surveyed wells. Data computed with a given time-depth chart are corrected by assuming a constant rate of change of velocity between two or three wells.

General Correction Schemes

A more general correction scheme is to compare the time-depth chart used for the preliminary computations with each of the surveyed wells. For a certain arrival time, say two seconds, there will be at each well a discrepancy between the computed depth and the depth from which a two-second reflection actually comes. These discrepancies can be set down on a map of the area and contours can be drawn for them; then at any point the error due to using the time-depth chart can be evaluated on the basis of all available velocity information.

The obvious difficulty with the method is that a correction map will only do for one arrival time. In practice it would be necessary to make correction maps for several arrival times—perhaps one for every tenth of a second—and to interpolate between the maps. Such a procedure would be too cumbersome for routine work.

A practical way to avoid using numbers of maps, and particularly to avoid having to interpolate between them, is to consider what horizons are likely to be contoured in an area and to make correction maps for certain *horizons* instead of certain arrival times. If the depth of an interesting horizon is known in all the surveyed wells, the arrival time of a reflection from it can be found from the velocity data for each well. The false depth is then read from the preliminary time-depth chart, and the depth correction at each well computed. These depth corrections are set down on a map of the area and then contoured just as before. The resulting correction map will be immediately applicable to the preliminary contour map of the seismic horizon in question; it gives a direct additive depth correction. A seismic map corrected according to this scheme will be accurate at the wells and as good elsewhere as the available information permits. Another such map can be drawn for any horizon which can be recognized in the wells.

It may be objected that nothing is really known about the position of the horizon between the wells, and that the correction is therefore mere speculation. However, this objection holds for any correction scheme based on well velocities. It is no worse to assume that the corrections for an *horizon* change continuously between wells than to say those for an *arrival time* do the same thing. The horizon is supposed to be continuous except for faults—and faults undoubtedly introduce discontinuities in the corrections for arrival times as well as for horizons.

Sample Contouring Horizons

Three stratigraphic markers that are of interest to oil prospectors in the San Joaquin Valley have been chosen as sample contouring horizons. They are as follows: the top of the Rio Bravo Sand (equivaland), the top of the Olcese Sand (equivalent), and a marker in the cherty shale above the Stevens-Greeley Sand. These markers were chosen because of their nearness to producing zones and because they can be correlated in wells but they do not necessarily reflect seismic energy. In fact, continuous reflections from any horizon are rare in the area, but the correction scheme applies as well to phantom horizons as to actual reflectors. Figs. 7 and 8 show contours of the depth and average velocity to the marker above the Stevens, and Figs. 9 and 10, and 11 and 12, the same things respectively for the Olcese and the Rio Bravo.

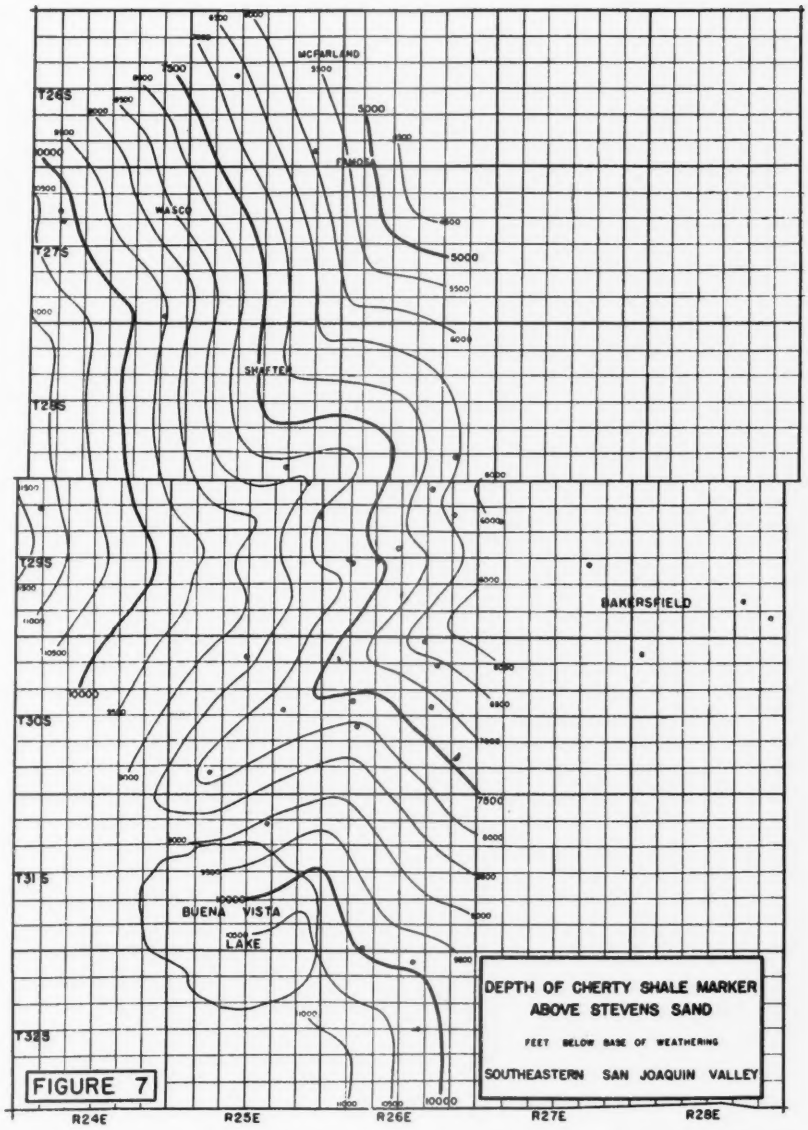

FIG. 7. Depth from base of weathering to cherty shale marker above Stevens sand.

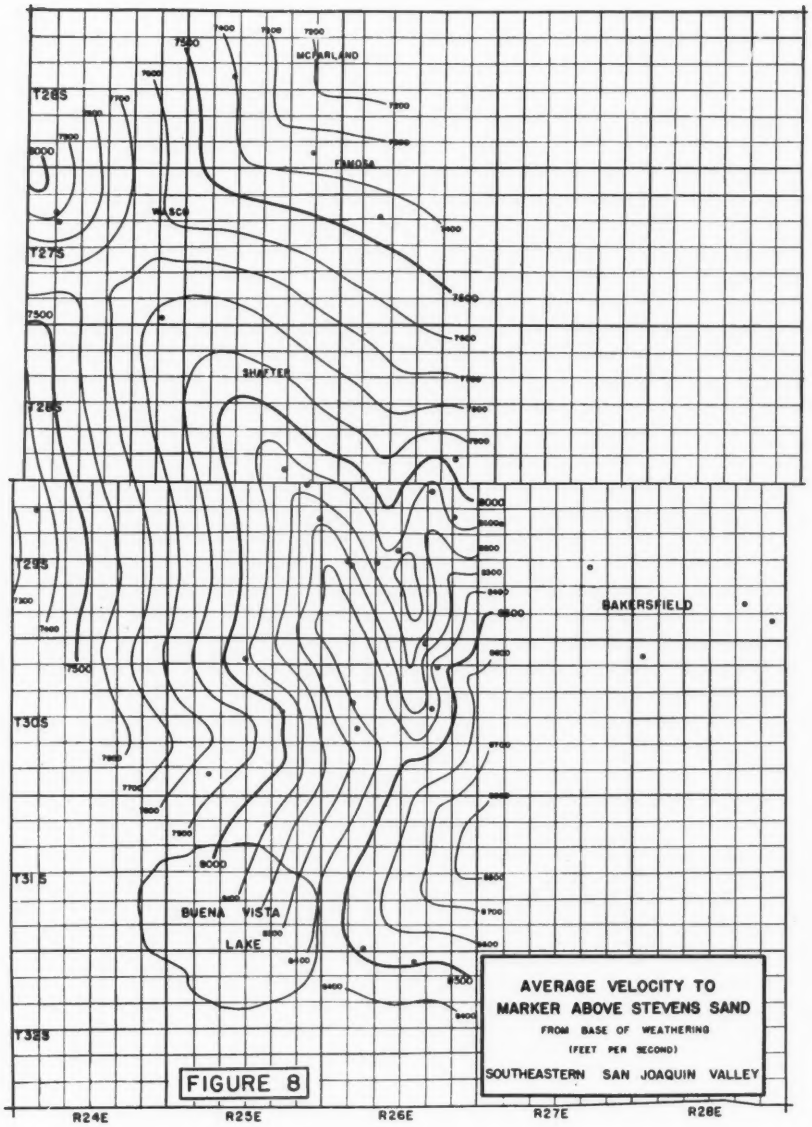

FIG. 8. Contours of average velocity to cherty shale marker above Stevens sand

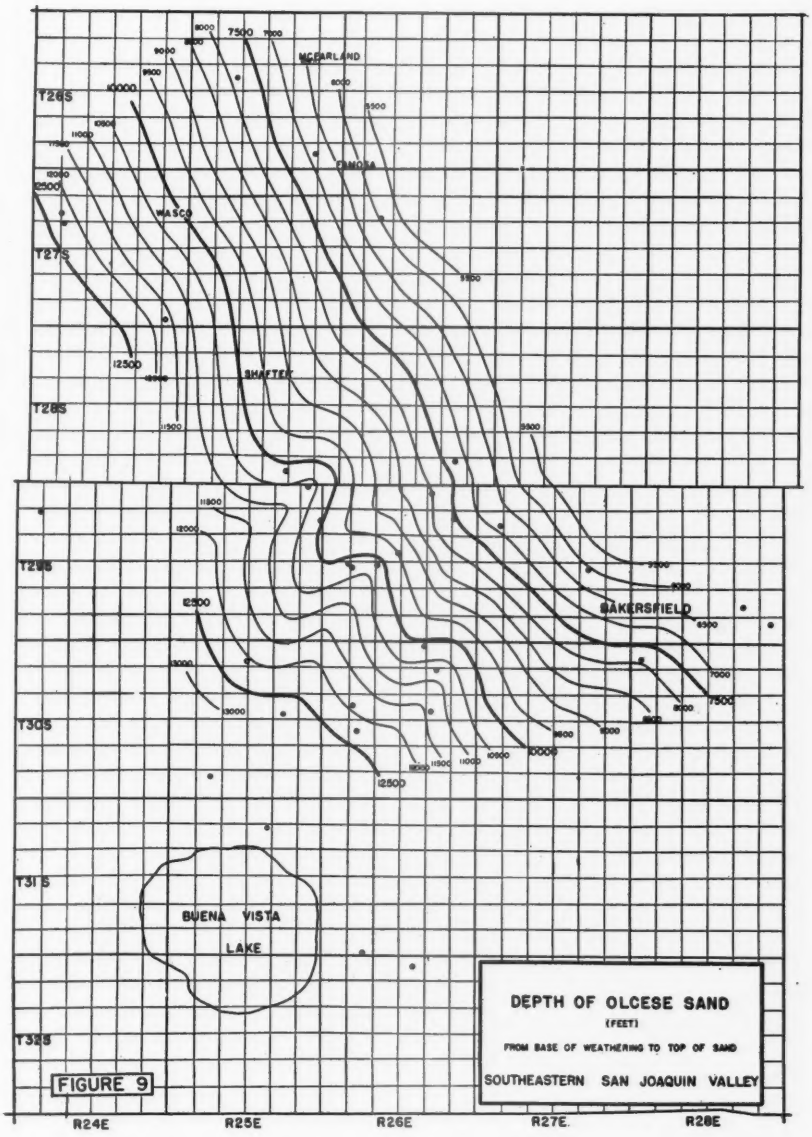

FIG. 9. Depth from base of weathering to Olcese sand.

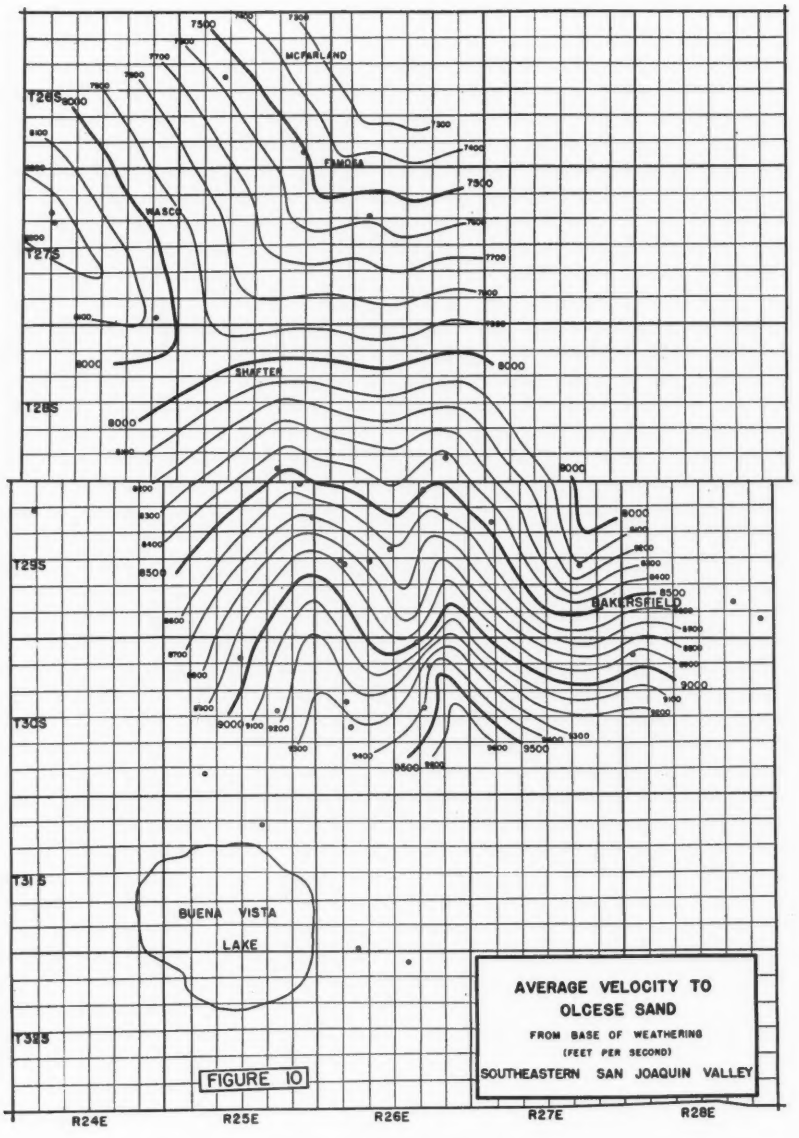

FIG. 10. Contours of average velocity to Olcese sand.

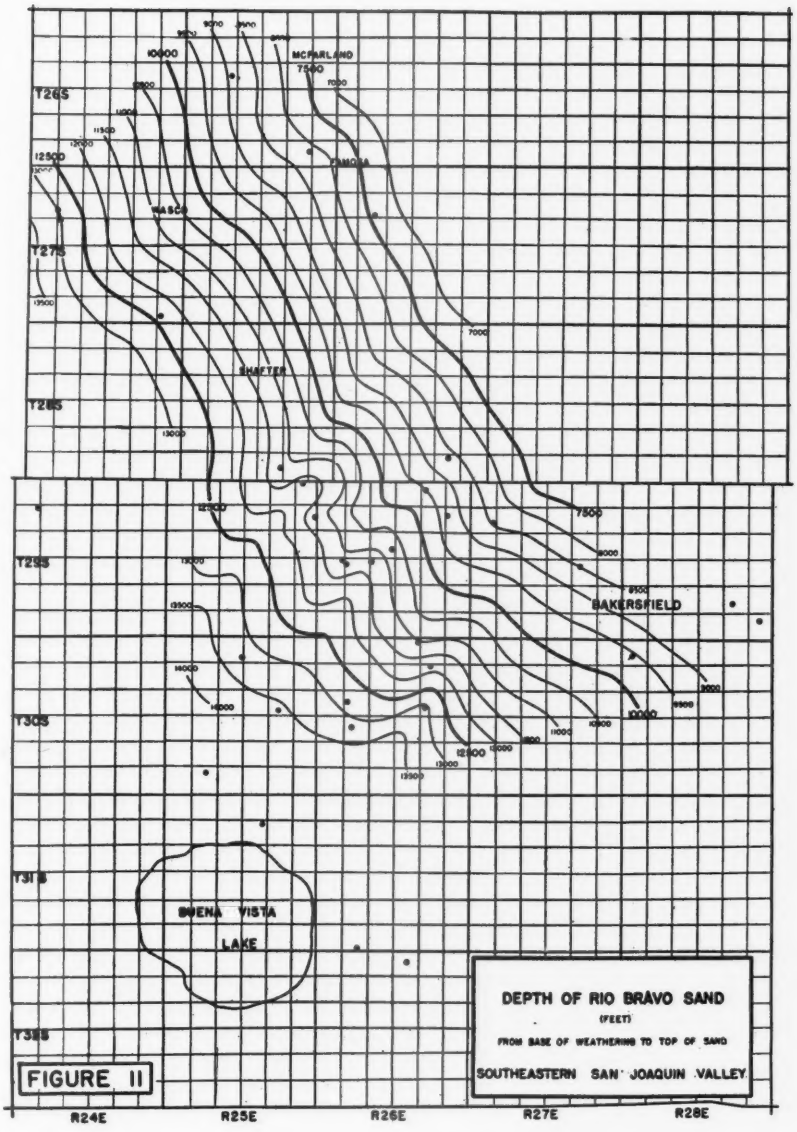

FIG. 11. Depth from base of weathering to Rio Bravo sand.

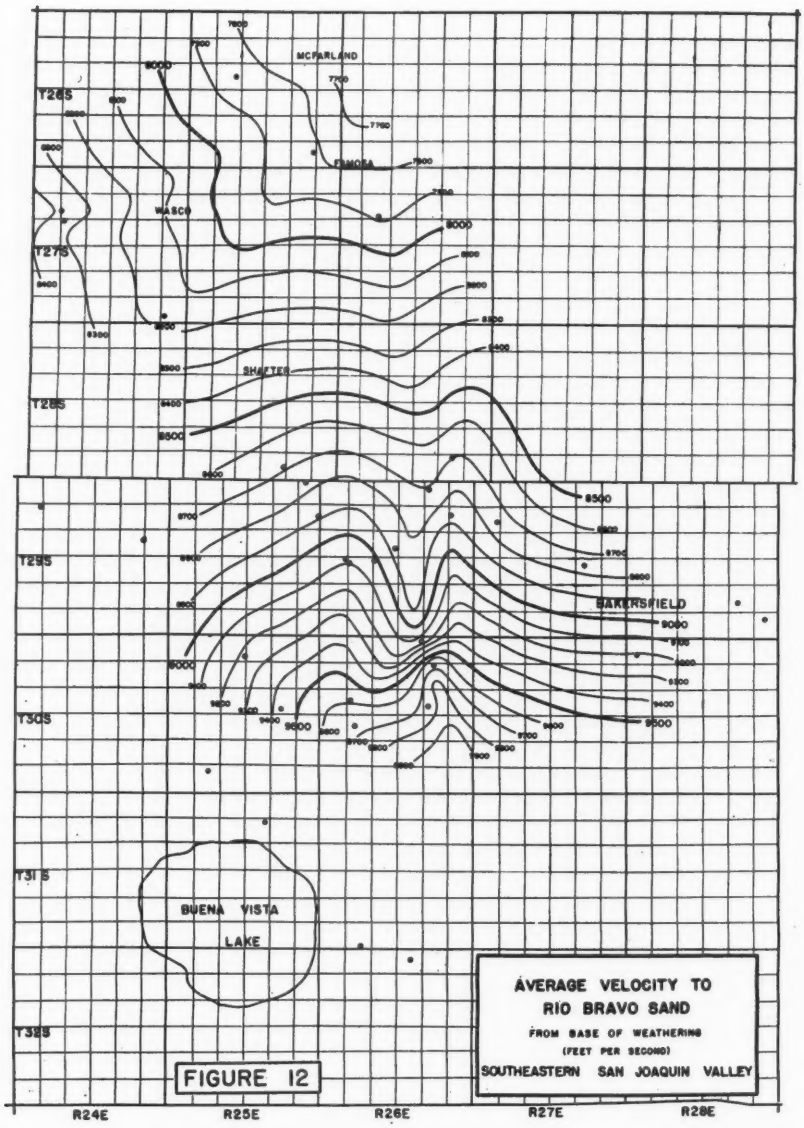

FIG. 12. Contours of average velocity to Rio Bravo sand.

These contour maps demonstrate three general facts. First, stratigraphy is not the dominant factor controlling velocity; if it were, the average velocity to a marker would be roughly constant or decrease slightly with depth. Second, loading is not the dominant factor either

for if it were the depth contours and the velocity contours would be roughly parallel. Third, while the maps show similar zones of high, medium, and low velocity, there are large-scale velocity anomalies

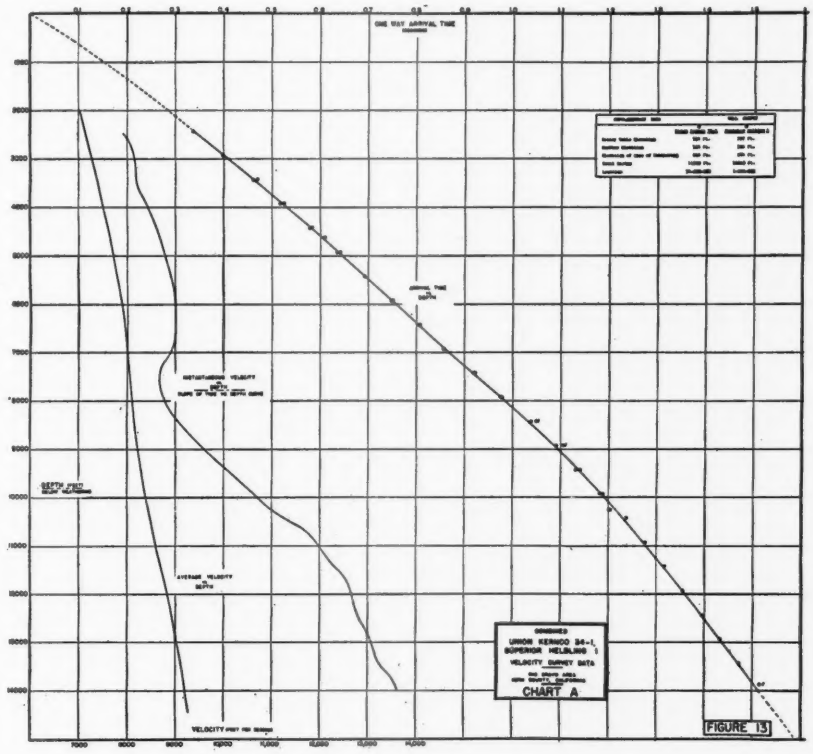

FIG. 13. Chart A. Time-depth chart based on velocity data from two wells—Union Kernco #34–1 and Superior Helbling 1.

which do not have any obvious cause or regularity. The inference is that lithology—especially local variations in lithology—is comparable to stratigraphy and loading in its influence on velocity.

Sample Correction Maps

For drawing sample depth-correction contours, a preliminary time-depth curve, typical of the area, was chosen for illustrative purposes. Data for two centrally located wells, the Superior Helbling #1 and the Union Kernco #34-1 were combined (see Fig. 13) to form a time-depth chart, which will be called "Chart A" for the sake of brevity. If Chart

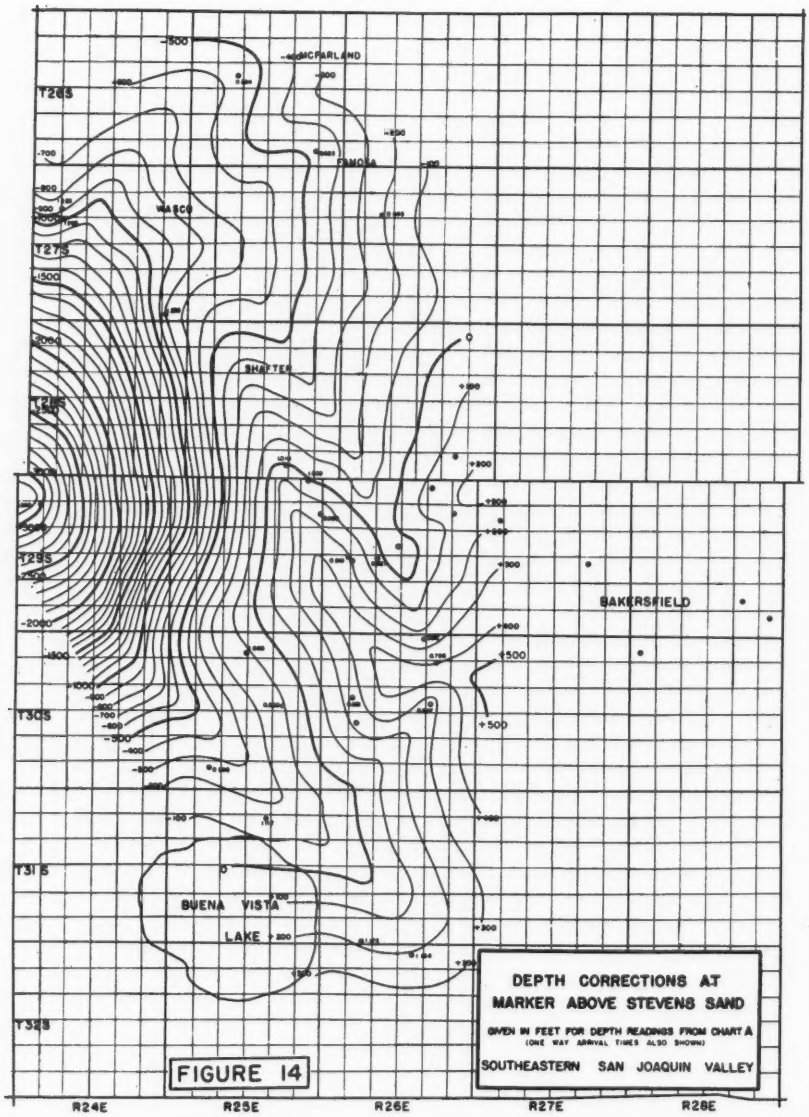

FIG. 14. Contours showing corrections in feet for depth readings made from Chart A (Fig. 13), on marker above Stevens sand.

A is used to compute seismograph maps on the horizons used as examples, the corrections that should be applied to the computed depths are shown in Figs. 14, 15 and 16. The size of the errors that would be

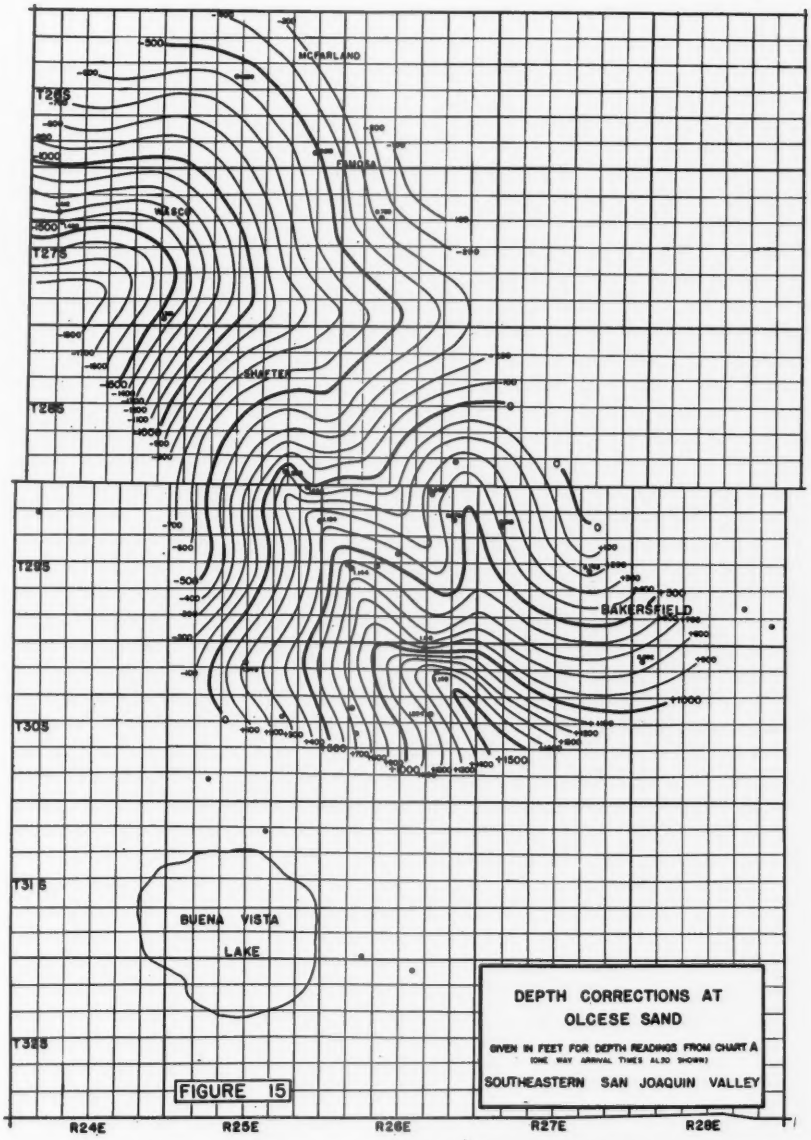

FIG. 15. Contours showing corrections in feet for depth readings made from Chart A on Olcese sand.

made in the preliminary maps, and the practical importance of making the corrections, are illustrated by the fact that on each of the maps the contour values change by *more than three thousand* feet.

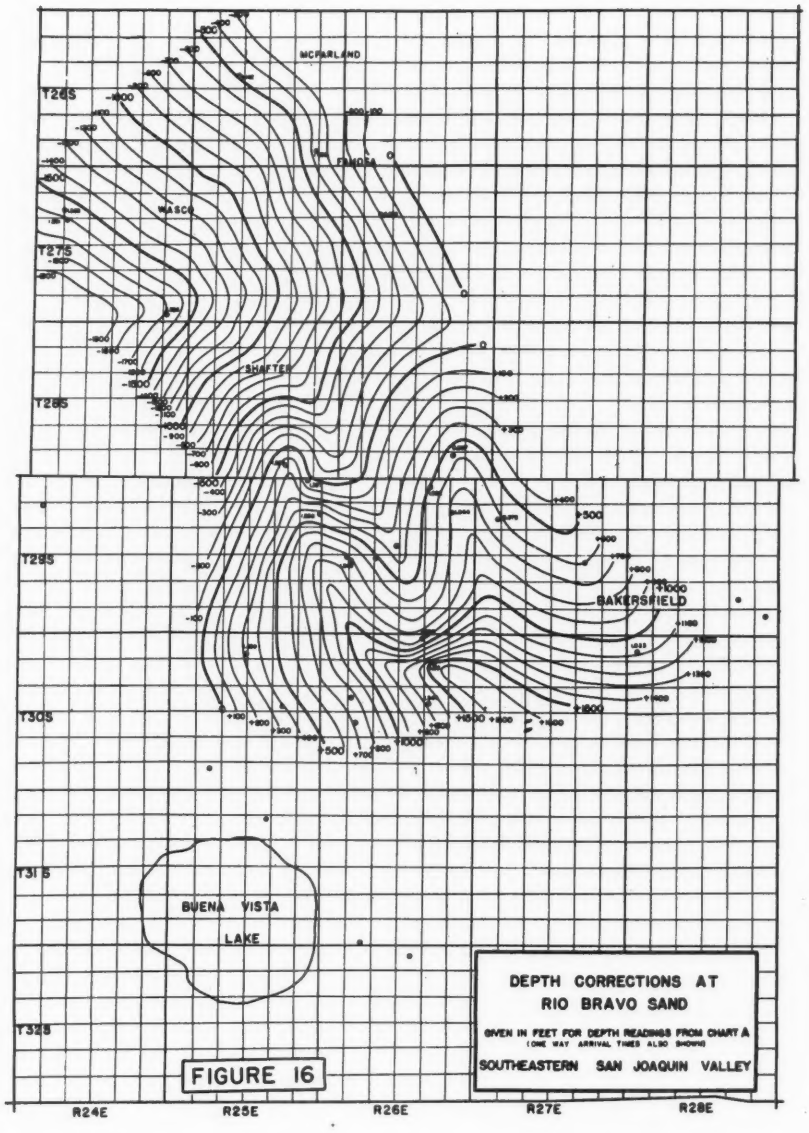

FIG. 16. Contours showing corrections in feet for depth readings made from Chart A on Rio Bravo sand.

Errors in Depth Corrections

A detailed study of typical errors in the construction and use of depth correction charts is beyond the scope of this paper, but the effects of these errors can be summarized as follows:

1. If a geological marker is miscalled by about sixty feet in a well, the area around the well will be wrong by about five feet.
2. Errors in the well-velocity measurement in feet per second result typically in errors of about the same number of feet in the depth correction.
3. Study of the known detailed variations in the maps shows that contours wrongly placed through lack of well control may cause local variations of fifty or a hundred feet. These are the largest and least controllable errors that will be encountered, and there is no remedy for them but closer velocity control.

Direct Computation Schemes

The ways of taking advantage of velocity data that have been considered so far are devices for correcting a map computed with a single time-depth curve. Another approach to the problem is to compute the results directly from velocity maps, without any preliminary computation. All direct methods suffer, however, from the basic difficulty that there are too many variables for convenient handling, at least in field work. Direct computations for so many variables require interpolations between maps, which is a tedious process, and for routine seismic computing it is almost necessary to reduce in one way or another the number of these variables. This can be done by preparing time-depth charts along profiles which are to be shot, or as Olson[1] proposes, reducing the well velocities to analytical functions of similar form and calculating algebraically instead of with charts.

VELOCITY AND STRATIGRAPHY

Attacks on the seismic velocity problem lead naturally to the question of the relation between velocity and stratigraphy. The most important influence on velocity is of course depth, and the depth gradients of velocity have been given much comparative study. When it appears that velocity dependent on depth alone will not do for seismic

[1] W. S. Olson of The Texas Company presented a paper on the same subject as this one to the Pacific Coast Section of the A.A.P.G. in November, 1940. Mr. Olson's paper included Figs. 3, 7, 8, 11, 12, 14 and 16 of this paper, and they are here published with his permission.

computations, the next step is clearly to try to establish a connection between velocity and the geologic section. If such a connection could be set up it would be of great practical importance, for after three or four wells in a geologic province had been shot for their velocities, depth correction maps for the whole area could be constructed from no other data but the logs of wells. The trouble and expense of shooting more wells could be avoided, and not only wells which could be shot but every logged well would be a control point.

Qualitative Observations

The outlook at first is fairly promising. Though the maps previously shown demonstrate that the velocity variations are not simple, it is common experience that at a given depth older or harder rocks have higher velocities, and that average velocities to a certain depth decrease down dip as younger formations occupy a larger proportion of the section. To study the matter in more detail it is necessary to consider the velocity contributions made by the different components of the section, and to use the interval velocities of these components. Fig. 17 shows the so-called instantaneous or local velocities in representative wells, as curves based on observed interval velocities. It appears, first, that the interval velocity just above the Rio Bravo increases with increasing depth. Second, there seems to be a low value of the velocity some two or three thousand feet above the Rio Bravo, and then a high value fifteen hundred or so feet higher. While these facts are only qualitative, they are encouraging.

Velocities of Stratigraphic Components

To deal with the problem systematically, the easiest course is to divide the section up into stratigraphic components of suitable thickness and attempt to assign a velocity to each. It is clear that the velocity of each component will be different for different depths, so a curve of velocity against average depth of the geologic interval might be drawn on the basis of all surveyed wells. Then the average velocity near an unknown well could be computed by reading from the curves the velocities of the geologic intervals in the well and combining them suitably. The test of such a method would be simply to draw the curves, apply them to a known well, and see how the computed average velocity of time-depth relationship checks with the observed one.

For the purpose in hand, we divide the section up into the following intervals, chosen mainly because they can be correlated in the wells:

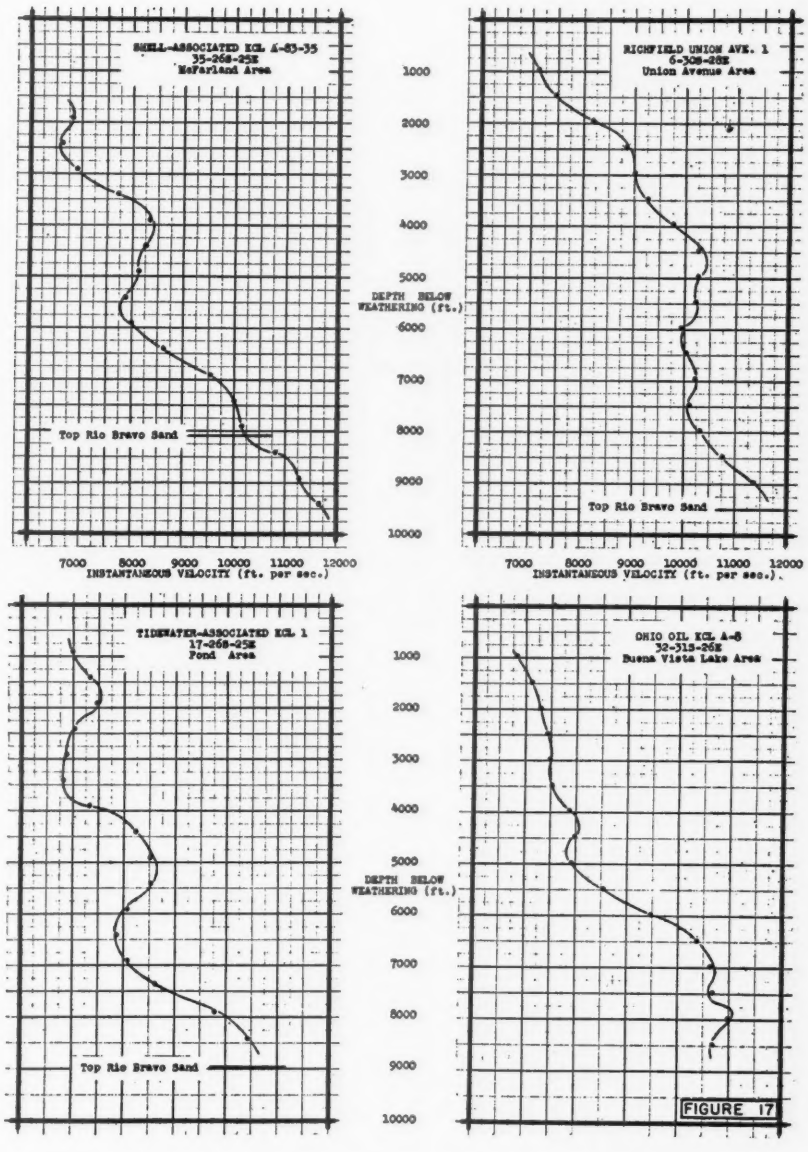

FIG. 17. Apparent velocity distribution in section of four representative wells based on observed interval velocities.

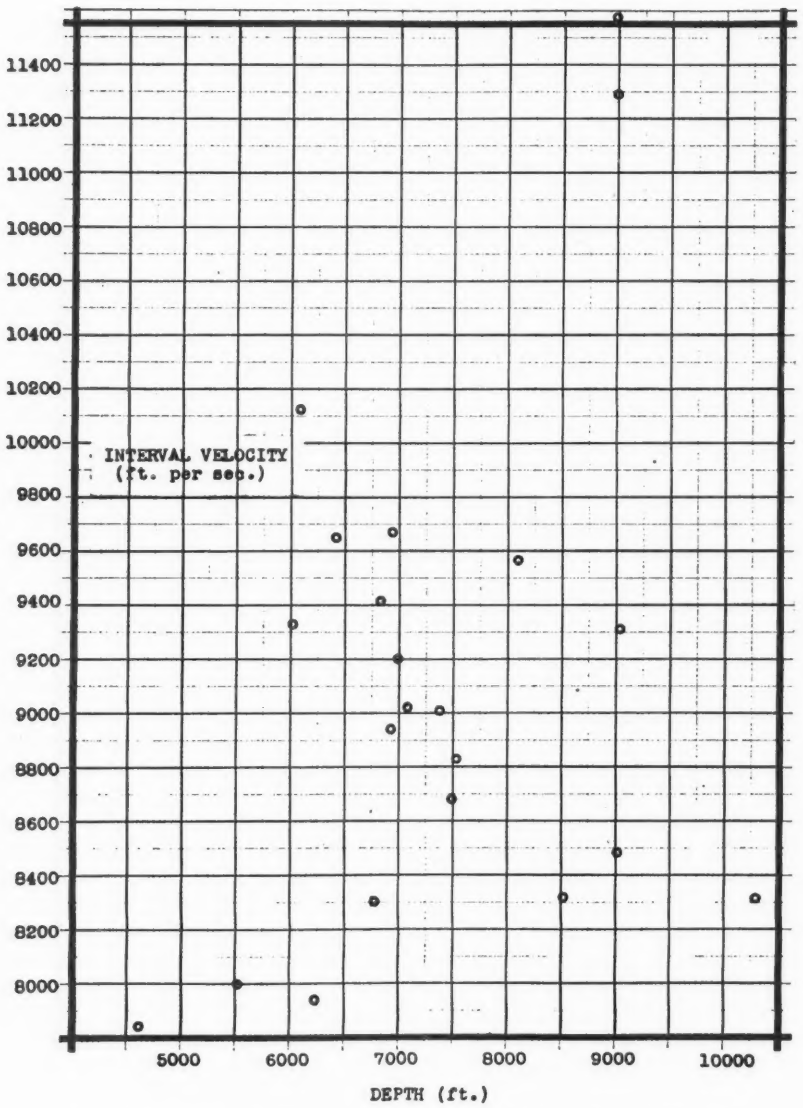

FIG. 18. Interval velocity vs. depth of mid-section from base of Pliocene sands and silts to marker above Stevens sand.

1. Base of weathered layer to base of Pliocene sands and silts.
2. Base Pliocene to a marker in the cherty shale above the Stevens-Greeley sand.

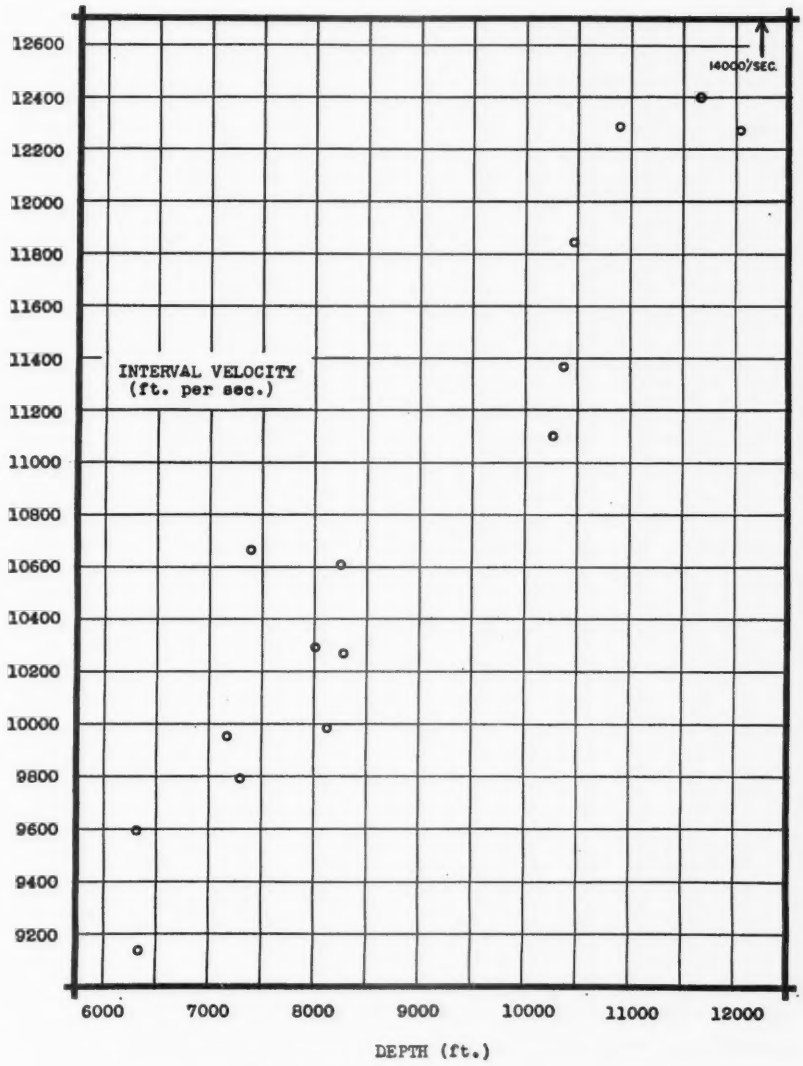

FIG. 19. Interval velocity vs. depth of mid-section from top to bottom of Olcese sand.

3. Cherty shale marker to base of Stevens-Greeley.
4. Base of Stevens-Greeley to top of Olcese sand.
5. Olcese sand.
6. Base of Olcese to top of Rio Bravo sand.

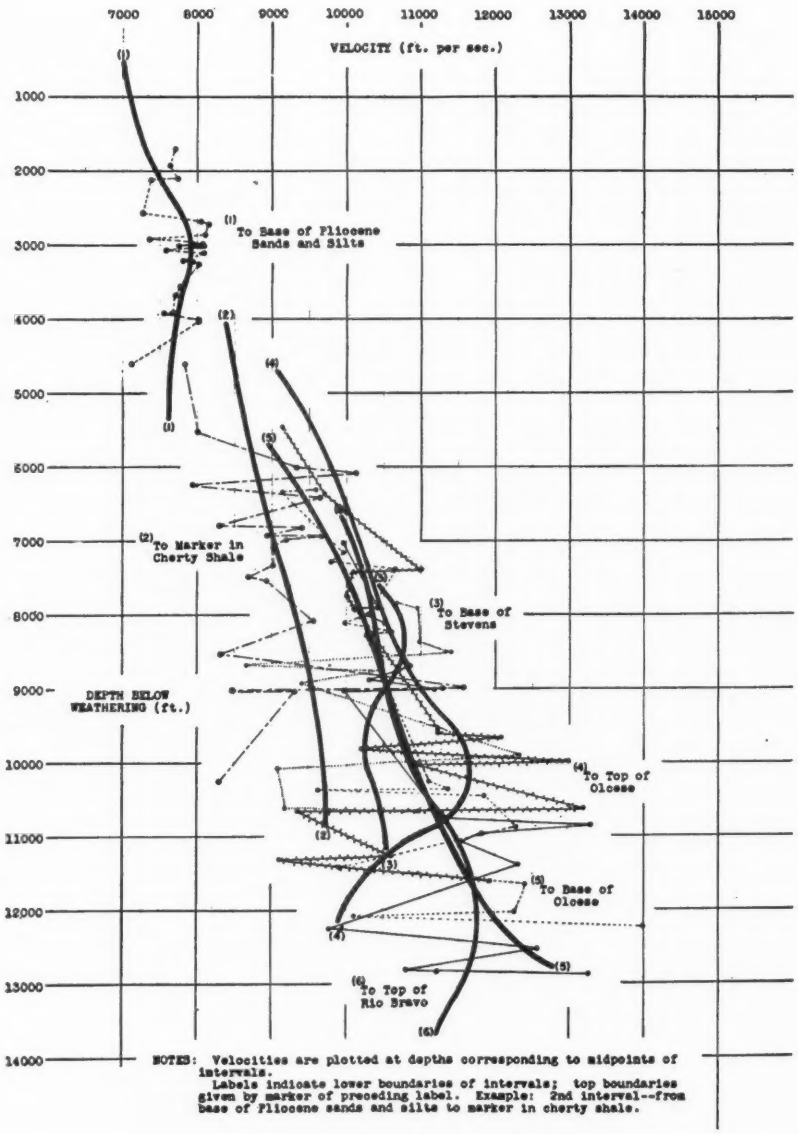

FIG. 20. Composite graph of interval velocity vs. depth. Velocities are plotted at depths corresponding to mid-points of intervals defined by:

1. Base of weathering to base of Pliocene sands and silts;
2. Base of Pliocene to marker in cherty shale above Stevens;
3. Cherty marker to base of Stevens;
4. Base of Stevens to top of Olcese;
5. Top of Olcese to base of Olcese;
6. Base of Olcese to top of Rio Bravo.

There is not complete agreement among specialists in the area as to the actual boundaries of these intervals in all the wells, but the errors introduced by wrong choice of boundary would not be large enough to vitiate the conclusions.

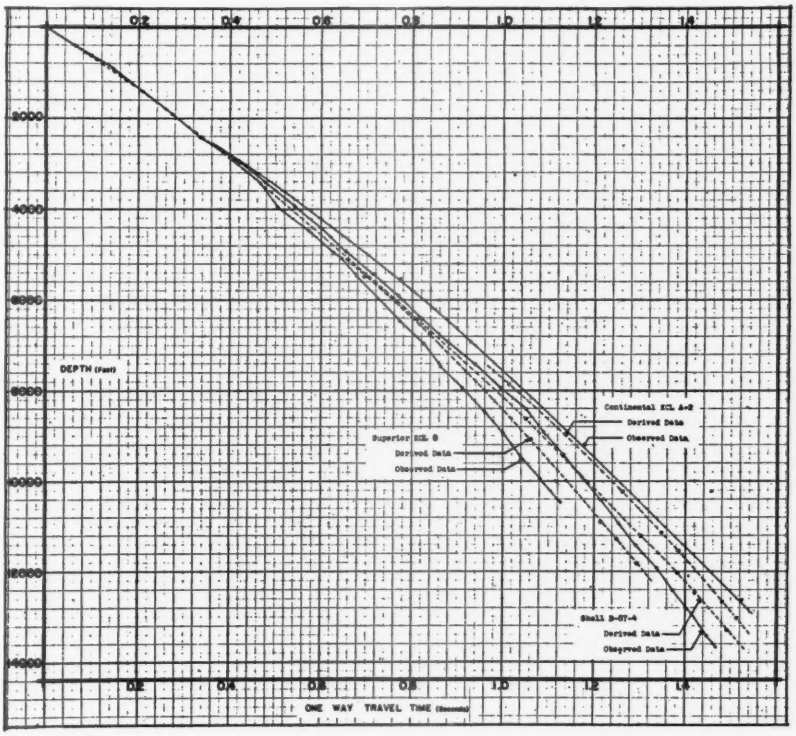

FIG. 21. Comparison between observed time-depth curve and curve derived from graphs of Fig. 20 for three typical wells.

Figs. 18 and 19 show interval velocity vs. depth of interval taken from various wells from the intervals 2 and 5 above. The points on Fig. 18 are scattered pretty much at random, and give little hope that any systematic relationship with depth can be found for that interval. Fig. 19 is a little more encouraging; the points do seem to line up, in a rough way. Fig. 20 is a composite graph of interval velocity vs. depth for all the intervals, and represents all the usable data in the area under discussion. For each interval the points are plotted and a sort of generalized curve is drawn in heavy lines to represent

the trend; however, the points are so badly scattered that it is not possible to say that there is a functional relationship between the depth and the velocity.

Test of Depth-Stratigraphy Method

Fig. 21 shows for three typical wells the comparison between the observed time-depth curve and the time-depth curve derived from the graphs of Fig. 20. The discrepancies in depth for a constant arrival time run from two to six hundred feet. When these discrepancies are compared with the depth differences shown on Fig. 3 they seem to be small; it might be concluded that the depth-stratigraphy method of computing velocities, while not a success, was at least better than using a single time-depth relation over the whole area. Actually the velocities can probably be computed as well by interpolation from maps if there are enough data in the area so the method can be applied.

It can be argued that the above test may not be conclusive, because the points used to plot the curve of interval velocity against depth might be more accurate if the wells were shot at shorter depth intervals. However, it is difficult to imagine that discrepancies in the interval velocity as large as three or four thousand feet per second would exist as a result of not shooting in the wells at short enough intervals, simply because the velocities for the individual wells hardly vary that much from top to bottom. It follows then that at a given depth a certain stratigraphic interval may have widely different interval velocities. This is probably due to differences in lithology, and suggests the next approach to the problem.

Velocities in Sands and Shales

A second attempt to predict well velocities from their logs was made by assuming that there was a functional relation between interval velocity and depth for shales and a similar one for sands. The electrical logs were compared with the instantaneous velocity graphs of the wells (see Fig. 17) and tentative curves of interval velocity vs. depth for sands and shales arrived at statistically. A theoretical time-depth curve was then drawn for an actual well, and the discrepancies observed to be about four hundred feet, which indicates strongly that the electrical-log lithological classification is not refined enough to give the desired results.

CONCLUSIONS

Well velocity surveys show that lateral velocity variations in the San Joaquin Valley are serious enough to require systematic correction

of seismic data. Because of the number of variables involved, the easiest way to make such corrections is with correction maps for certain horizons.

All attempts made to date to relate velocity to stratigraphic position fail. This may be due to the coarseness of the well velocity data, but is more likely to be the fault of lateral lithologic changes in stratigraphic units. Advancement of the solution to the problem depends on better seismic data and on more detailed lithologic classification within stratigraphic units.

No data are presented to show whether the large variations in the area being studied are representative of other areas, but the possible consequences of such changes to seismic prospecting are important enough to make their evaluation in new prospects a necessary step.

The author wishes to acknowledge his debt to Eugene Vallat of the Continental Oil Company, who not only contributed ideas but furnished the geological information on which the entire paper is based. The velocity data were collected by the California Well Velocity Association. Cecil H. Green of Geophysical Service Inc. motivated the work, and Frederick Romberg of the same organization criticized it in detail.

THE PROBABLE ERRORS OF DELTA-*T* VELOCITIES*

FREDERICK ROMBERG†

ABSTRACT

It is proposed to test the accuracy of delta-*t* velocities by finding the average deviations and probable errors of the delta-*t* readings. Data are provided to demonstrate that errors smaller than one per cent can reasonably be expected. The quantitative effect of numbers of observations, shooting set-ups, quality of data, and computing methods are discussed, and examples shown. Examples of regional and local lateral gradient are added.

INTRODUCTION

It is well known that the average velocity of a reflected seismic impulse can be found by observing the lag in its arrival time as the distance between the shot point and the recording point increases.[1] The accuracy of such determinations depends on the shot point-recording point distance, or spread, and on the amount it is increased. The greater the spread, and the greater the difference between the smallest and largest spread, the more accurate the velocity. In practice the required accuracy is attained in two ways. The first is to use an unusually large spread and take only a few readings; this is the velocity-profile method.[2] The second, the so-called Δt-method, is to read time-lags from routine reflection records, and to make up for the smallness of the spread by taking a large number of observations. The worth of velocities computed from Δt observations varies widely, so that it is important to have some kind of a practical test for finding out how good the velocities really are. This paper presents data to which such a test has been applied; it uses the apparent probable errors to appraise the computed velocities and to suggest limitations for the Δt method. The average velocity is computed from the observed time lag with the equation[2]

$$v^2 = \frac{\Delta x[2x + \Delta x]}{\Delta t[2t + \Delta t]}, \qquad (1)$$

* Presented at the Annual Meeting, Houston, Texas, April 2, 1941.

† Geophysical Service, Inc., Dallas, Texas.

[1] Eugene McDermott, "The Application of Seismography to Geological Problems." Bull. A.A.P.G. V. 15, No. 11, Nov. 1931.

[2] C. H. Green, "Velocity Determinations by Means of Reflection Profiles," GEOPHYSICS, V. III, No. 4, Oct. 1938.

[3] Whittaker and Robinson, "The Calculus of Observations," p. 188.

where v = average velocity
Δt = time lag
t = arrival time
x = shorter spread
Δx = difference between shorter and longer spreads.

Any set of records in which x and Δx are constant can be used for finding a set of average Δt's. The arrival time range is arbitrarily divided up into components—for instance, of 0.1 second apiece—and an average t and Δt found from all the observations in the chosen component. It may be proper for the components to overlap or to vary in size; this will be discussed later. The average arrival times are usually reduced to depths to give velocity-depth data, according to the equation

$$z = vt \cos \theta, \quad \text{where} \quad \theta = \sin^{-1} \frac{x}{vt}. \tag{2}$$

The data can then be translated into zero-spread time-depth data with

$$t_0 = z/v. \tag{3}$$

Velocities computed from Equation 1 are subject to various errors. First, refraction and other effects may invalidate the equation, to a greater or less extent. Second, there are systematic errors in the arrival time t, because t is ordinarily read a cycle or two behind the first appearance of the reflections, and the size of these cycles changes with the reflection frequency. Third, and most important, there is an error in Δt. The influence of such an error, $\delta \Delta t$, on the computed velocity is found by differentiating (1) with respect to Δt. The result is that

$$\delta v = - \delta \Delta t \frac{v}{2} \left[\frac{1}{\Delta t} + \frac{1}{2t + \Delta t} \right]. \tag{4}$$

(Equation 4 is of course not accurate for large errors, since v is not a linear function of Δt, but it is perfectly good as an approximate index for the worth of a velocity. There is no need to determine probable errors accurately.) Since Δt is small compared to $2t$, the term in brackets can be approximated by $1/\Delta t$, and we have

$$\delta v = - v \frac{\delta \Delta t}{2 \Delta t}. \tag{4a}$$

The effect of the error δv on the computed depth is found approximately by differentiating the z in (3) with respect to v. The result is

$$\delta z = t_0 \delta v. \tag{5}$$

ERRORS IN Δt

The errors in individual readings of Δt are due to dips of the reflectors, irregularities in the weathered layer, frequency changes resulting from increasing spreads, and the ordinary instrumental uncertainties in reading times from the records. These can be divided up into systematic and random errors. Errors due to frequency changes will probably be systematic, and can be checked only by comparing the computed velocities with well velocities or with velocities taken from different spreads. Errors from the other sources ought all to be at random. Errors from time-reading uncertainties are clearly random errors; if there are enough shot points and they are suitably distributed (for example, if the points are all shot in two directions), the dip and the weathering errors will be random also. These random errors can be dealt with by assuming that they occur in a Gaussian distribution and computing probable errors for them.[3] The probable errors are the best available indices of the relative worth of an average Δt, and consequently of the worth of the derived velocity. The probable error Q_n of an average is found from

$$Q_n = 0.675 s/\sqrt{n}, \tag{6}$$

where s is the standard deviation of the set of observations and n is their number. The standard deviation can of course be computed from the definition

$$s^2 = \sum_{i=1}^{i=n} \frac{[\overline{\Delta t} - \Delta t_i]^2}{n - 1}, \tag{7}$$

where the Δt_i's are the individual readings and $\overline{\Delta t}$ is their average. However, for symmetrical distributions some work can be avoided by finding an approximate value for s from the average deviation D, as follows:

$$s = \sqrt{\pi/2}\, D. \tag{8}$$

D is computed approximately from

$$D = \overline{\Delta t} - A \tag{9}$$

where A is the average of all the Δt_i's *below* $\overline{\Delta t}$.
Equations 6, 8, and 9 are now combined to give

$$Q_n = 0.85[\overline{\Delta t} - A]/\sqrt{n} \tag{10}$$

for the probable error of an average Δt.

Equation (4a) can be written

$$\delta v/v = -Q_n/2\Delta t. \tag{4b}$$

The size of Δt is of course mainly determined by the quantity $\Delta x(2x+\Delta x)/2t$ (see Equation 1), which is determined by the shooting set-up and the depth. The percentage of error in the velocity, then, depends on four indices: first, the arrival time; second, the set-up; third, the number of points shot; fourth, the average deviation of the Δt's. The set-up and the number of points can be changed at the discretion of the prospector. The average deviation, though, is a function of the quality of the data, and can be changed only by improving the method of taking readings.

APPLICATION TO OBSERVED DATA

The previously outlined theory should enable the seismograph prospector to attack the fundamental problem of how useful Δt velocities actually are. The crux of the matter is this: Will the deviations of Δt, in practice, actually be small enough to result in a usable velocity, considering the usual spreads and numbers of readings.

As a preliminary example, a set of Δt data from 144 locations in Jefferson Parish, Louisiana, has been computed. Table 1 shows the

TABLE 1

ΔT VELOCITY-DEPTH DATA, JEFFERSON PARISH, LOUISIANA

Depth z	*Velocity* v	*Error in Velocity* δv	*Error in* Δt $\delta\Delta t$	*Average Deviation*	*Number of Observations*
3,113	6,434	99	.0014	.007	8
3,757	6,626	62	.0007	.006	20
4,005	6,525	32	.0004	.004	45
4,477	6,682	44	.0004	.005	45
4,928	6,810	52	.0005	.005	35
5,455	6,991	42	.0003	.004	63
5,817	6,960	40	.0003	.004	59
6,336	7,140	61	.0004	.004	44
6,505	6,937	74	.0005	.005	35
7,116	7,163	56	.0003	.005	65
7,697	7,384	70	.0004	.005	62
7,970	7,292	97	.0005	.005	33
8,516	7,430	154	.0007	.007	30
8,482	7,093	154	.0008	.006	24
9,983	7,949	143	.0005	.004	25
10,550	8,090	137	.0004	.005	45
10,464	7,743	121	.0004	.005	52
11,319	8,077	194	.0006	.006	36
10,698	7,371	271	.0010	.007	18
10,954	7,285	238	.0009	.007	22
12,345	7,605	224	.0007	.007	32

tabulations and Fig. 1 the graph of the velocity-depth curve. It is to be expected that the average deviations, D, of the Δt readings will remain about the same whether the arrival times are small or large. The tabulations bear this out, showing an average deviation of about five or six thousandths for most of the range. This may be taken as

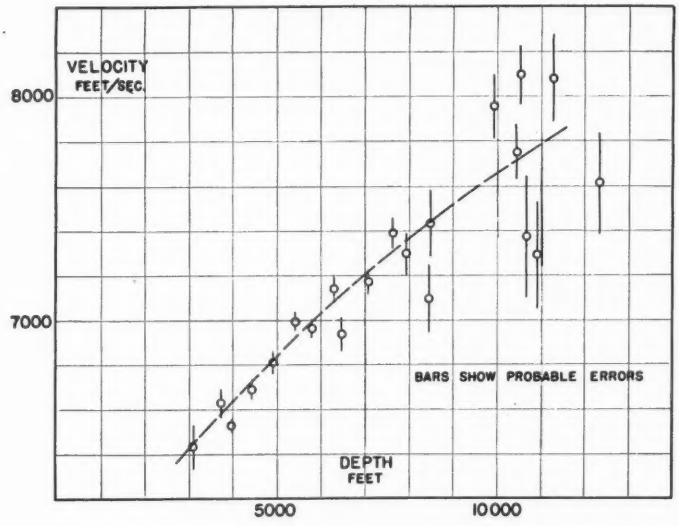

FIG. 1. Graph of velocity and errors computed from Δt data observed in Jefferson Parish, La.

an index of the quality of the observations, disregarding their number and their effect on the computed velocities. For instance, if the weathering differences or dips over a prospect show more variation than usual, the average deviations ought to be larger than on a prospect where dips are small and weatherings uniform.

The proportion of error in the velocity can be seen from Equation 4a to depend on the proportion of error in Δt. This proportion, $Q_n/2\Delta t$, increases rapidly with depth, since for great depths Δt becomes quite small. This is borne out by Fig. 1, where the probable errors (indicated by bars on either side of the depth-velocity points) increase rapidly with depth. Down to about seven thousand feet the errors are less than one per cent; deeper than that they are two or three per cent. The error of the curve in general (as distinct from the errors of the individual points) can be guessed at by observing that a smooth curve drawn through the points deviates from them, on the average,

about the amount of the probable errors. It may thus be assumed that the expected error in the velocity function corresponds in a rough way to the probable errors of the points. The practical conclusion is that the velocity down to about seven thousand feet is useful; below that it is not, in this example.

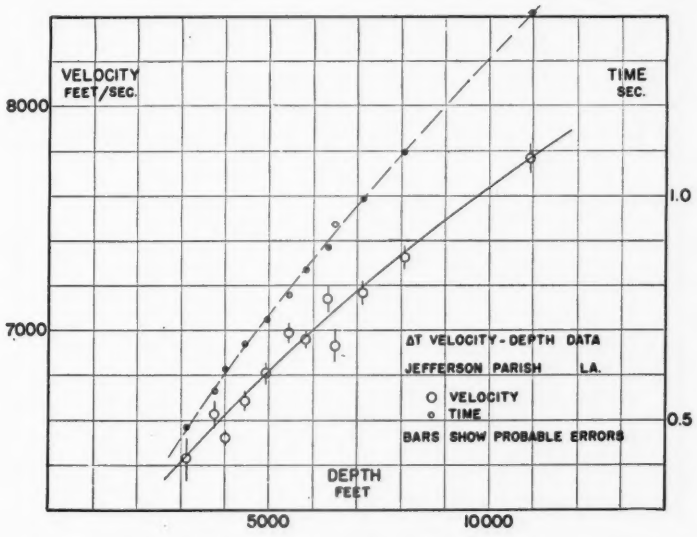

FIG. 2. Time-depth and velocity-depth data computed by combining certain points in Fig. 1.

COMBINATION OF *v-z* POINTS

Detailed examination of Fig. 1 shows that at depths greater than 10,000 feet there is no clear relation between the positions of the *v-z* points and the slope of the curve. This suggests at once that the points might be combined into an enlarged arrival time interval. An average t and $\overline{\Delta t}$ could be computed from all the observations in the new interval, and a new probable error found for the combination.

The question now arises as to whether the computed probable error will represent the uncertainty in Δt or the change in the average $\overline{\Delta t}$ across the interval. The answer is found by comparing the observed deviations of the Δt's with the estimated change in the average. If the deviations are sufficiently large compared to the change in $\overline{\Delta t}$, the new probable error will be mainly an index of the uncertainty.

In Table 1 it is seen that the total range of the $\overline{\Delta t}$'s below 10,000 feet is about .002 second, while the deviation is about .006. This indicates that it is quite safe to compute a probable error.

The old v-z points are combined in their proper weights—the easiest way is to find new averages for t and Δt directly from the tabulations. To avoid going back to the original data to compute Q_n, a formula for combining probable errors is derived by considering how to add standard deviations. If two sets of observations contain respectively a and b readings, have averages of A and B, and standard deviations of s_a and s_b, the standard deviation s_c of the combined observations is found from

$$s_c^2 = \frac{[a-1]s_a^2 + [b-1]s_b^2}{a+b-1} + \frac{aA^2 + bB^2 - [a+b]C^2}{a+b-1}, \qquad (11)$$

where C is the average of all the observations.

If the standard deviations are replaced by probable errors, according to (6), Equation (11) then becomes

$$Q_c^2 = \frac{[a^2-a]Q_a^2 + [b^2-b]Q_b^2}{[a+b]^2 - [a+b]} + \frac{0.455[aA^2 + bB^2]}{[a+b]^2 - [a+b]} - \frac{0.455C^2}{a+b-1}, \qquad (12)$$

which can be used to compute the probable error of a combination of averages. Both (11) and (12) can be used for combinations of more than two sets. Fig. 2 illustrates the result of combining the last seven points of Fig. 1, and the four preceding ones, into single points. It is clear that the data are much improved by the combination.

In general it will be profitable practice to average Δt's in combined and overlapped arrival time intervals. Since the average $\overline{\Delta t}$ moves rapidly while t is small and slowly when it is large, smaller arrival time intervals are used for shallow depths and large ones for great depths. As soon as the arrival time interval is chosen too large the probable errors begin rising, as a consequence of the last two terms in Equation 12. (Normally the difference of these terms is about zero, but when an attempt is made to combine two sets of observations which have widely different averages they become important.)

Fig. 3 shows a set of velocity-depth data from Terrebonne Parish, Louisiana. The lower curve shows the points derived from arrival-time intervals of 0.1 sec. The upper curve shows points derived from overlapped time-intervals of 0.2 sec. It is to be expected that the upper curve will be smoother, but it is worth noting that the probable errors

are in general smaller. To get the most out of a set of Δt data, the arrival time intervals should not be made so large that the probable error ceases to be less with increasing arrival time interval.

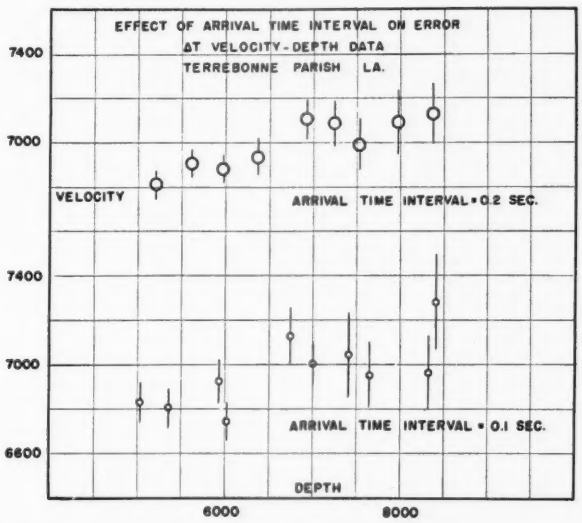

FIG. 3. Velocity-depth data showing the effect of using overlapped time-interval.

Fig. 2 also shows the time-depth values derived from the velocity-depth points, according to (3), and the depth errors, from (5). It is to be noted that the depth errors are proportional to the arrival times as well as to the velocity errors, and therefore so small as not to be worth considering at depths less than six thousand feet. This provides a good index of the usefulness of the data.

COMPARISON WITH OTHER VELOCITY METHODS

The probable errors in the examples so far presented have been of the order of one per cent at depths of eight or ten thousand feet. These are as small as can be expected of velocities from the best velocity profiles. In the author's experience, the probable errors in velocity profiles are perhaps twice as large, at such depths as eight thousand feet, as those in the Δt examples here shown. The errors in velocity profiles would be reduced rapidly if many readings were taken, but in practice many readings are not taken.

By comparison with the errors in velocities found by shooting in wells, the errors in Δt velocities seem somewhat large. If a well shooting velocity actually represents the reflection velocity, it should in practice be just as accurate as the arrival time of a reflection itself. This means that the probable error of a well velocity is one-fifth or one-tenth that of a velocity profile or an ordinary Δt velocity.

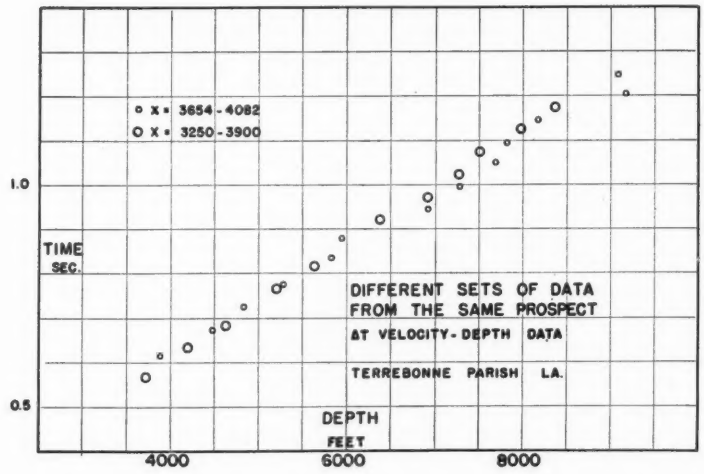

FIG. 4. Comparison of Δt velocities computed from records with different frequencies.

SYSTEMATIC ERRORS IN Δt DATA

The previous discussion of the errors in Δt velocities is all based on the assumption that there are no systematic errors in the observed Δt's. This assumption is of course false, but it is beyond the scope of this paper to examine quantitatively the sources of systematic errors. A rough idea of the importance of systematic errors may be got by comparing sets of data taken on a prospect under different recording conditions. Fig. 4 shows time-depth points observed with two different kinds of recording instruments on the same prospect. One variety of recording instruments was set up only in the marsh, and the other only on roads and high ground, with a resulting large difference in frequency between the two sets of records. It can be seen that there is not (except perhaps in the 7,000′–8,000′ range) any systematic difference in the points belonging to the two sets.

Fig. 5 shows velocity-depth data from the same prospect, this time taken with the same recording instruments and set-ups but with

different spreads. The velocity from the shorter spread appears to be the higher by 100–150′/sec. This is perhaps because the frequencies decrease with larger spreads, giving a too-large Δt and therefore a too-low velocity for the long spreads. In any case, the data in Figs. 4 and 5 may be taken to indicate that the systematic errors in the Δt's may be of the same order of magnitude as the random probable errors.

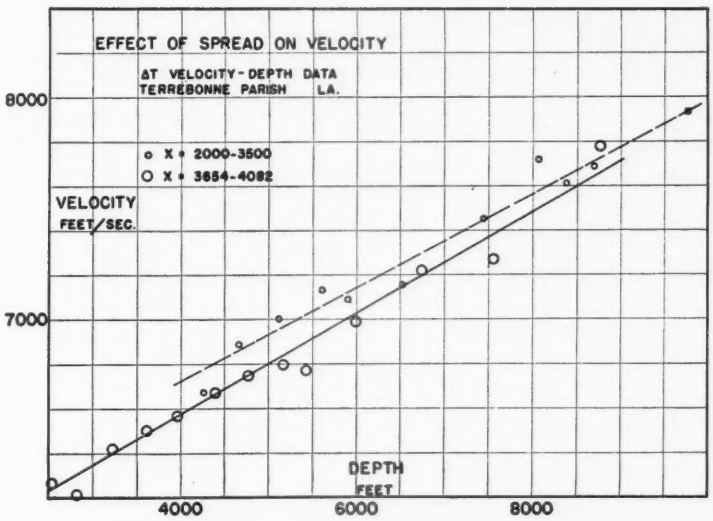

FIG. 5. Comparison of Δt velocities computed from records taken with different set-ups.

FACTORS INFLUENCING PRECISION

It has already been said that the precision of Δt velocities depends on four factors: the depth, the spread, the number of observations, and the average deviation. It is proposed to give examples of these effects on the computed velocities.

The most important of the factors is spread. Equation 4*a* shows that Δt should be as large as possible, and Equation 1 shows that the observed Δt will be roughly proportional to the sum of the greater and smaller spreads times their difference. Fig. 6 shows the computed velocities from two sets of data in Terrebonne Parish. In one of them $\Delta x(2x+\Delta x)$ is 1.44×10^6; in the other it is 4.32×10^6. The ratio of these two figures is about 3, which corresponds roughly to the relative lengths of the error lines in Fig. 6.

The next factor is the number of readings. Equation 10 shows that

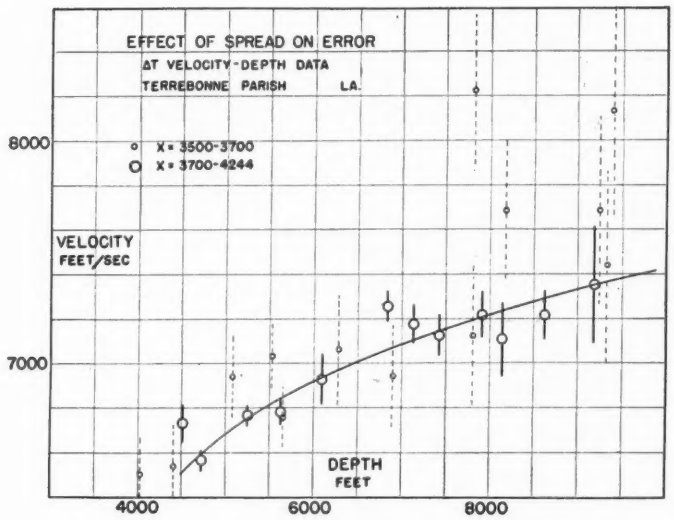

FIG. 6. Illustration of the effect of a change in spread on the errors in Δt velocities.

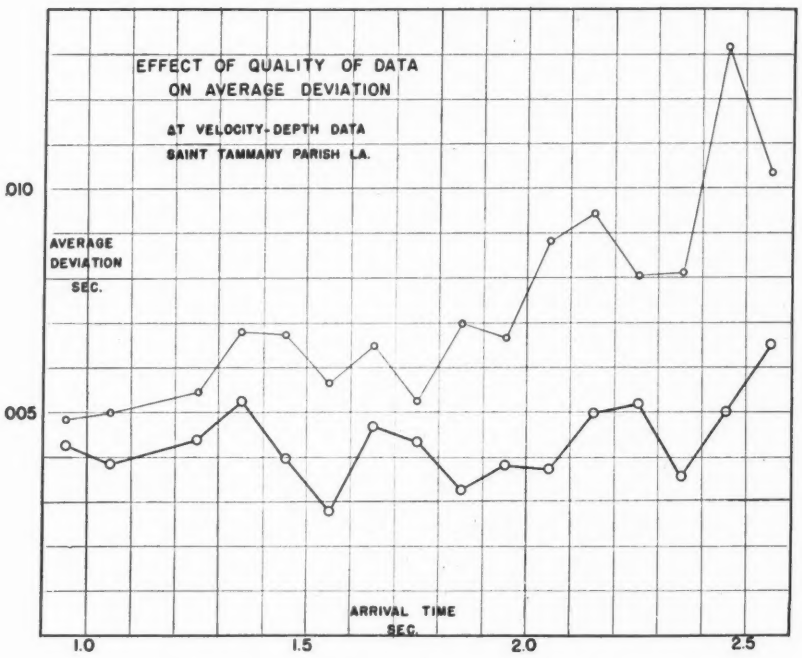

FIG. 7. Average deviations for good and bad data.

the error in Δt is proportional to the inverse square root of the number of readings; to halve the error quadruple the number of observations, etc.

The third factor is the average deviation. The errors in any set of Δt data are in direct proportion to their deviations. In general the magnitude of the deviation is determined by the record quality, the size of the irregularities in the weathering, and the prevalence of dip in the reflectors. Fig. 7 shows the deviations of two sets of Δt data from the same prospect. One consisted of records which were selected out of the group because they were shot along strike; the other was made up of what was left. It can be seen that the first set has deviations that are uniformly about half as large as those of the second set.

In general, then, it should be possible to estimate the velocity error for the velocity to an arrival time t for any particular set of data whose deviation is known. By combining (1), (10), and (4b), we have

$$\frac{\delta v}{v} = -\frac{0.85D}{\sqrt{n}} \frac{tv^2}{\Delta x(2x + \Delta x)}; \tag{13}$$

after estimating v (which has only to be done to ten or twenty per cent) we can readily discover how good a Δt velocity we may expect from a given set of data. Conversely, of course, if it is proposed to find a velocity with a given precision, it can be discovered how many observations will be necessary, or how wide a spread and spread difference will be needed.

LATERAL GRADIENTS OBSERVED FROM Δt DATA

Considering the current interest in lateral velocity gradients it is thought appropriate to include a couple of samples of Δt velocities observed in the same neighborhood, as demonstrating the size of the gradients to be expected in the Gulf Coast. Fig. 8 shows three time-depth curves computed from prospects in Terrebonne Parish, Louisiana, in a line and about eight miles apart. The two extreme prospects are indistinguishable, but the middle prospect varies so that reflections computed with a time-depth chart correct on the outer prospects would be in error by about five hundred feet. It can be seen from Fig. 2 that this is substantially more error than would be expected from the velocity uncertainty.

Fig. 9 shows an example of the regional gradient in Southern Louisiana. Time-depth curves from St. Tammany, Jefferson, and Terrebonne Parishes are plotted on one graph, and it is seen that,

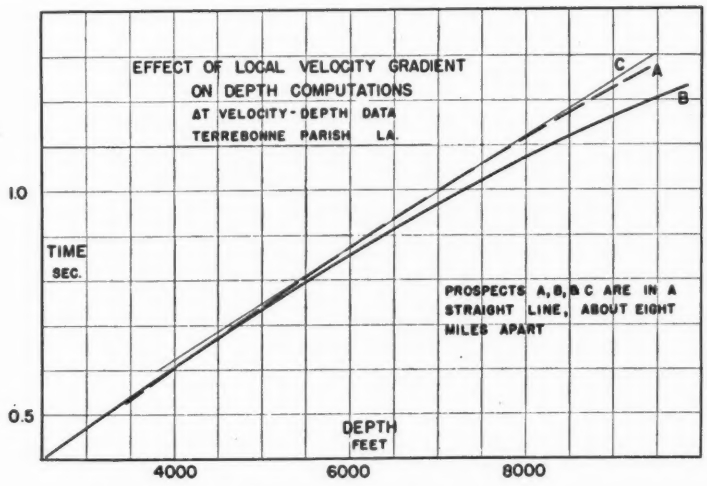

FIG. 8. Time-depth curves showing local velocity gradient.

while the curves are about alike down to 6,000′, they deviate from there on down. Actually, the regional variations in Fig. 9 are about the same size as the local variations in Fig. 8; this suggests some interesting speculations with regard to the local behavior of velocities in the Gulf Coast generally, which might change some of the seismograph pictures of the area in a startling way.

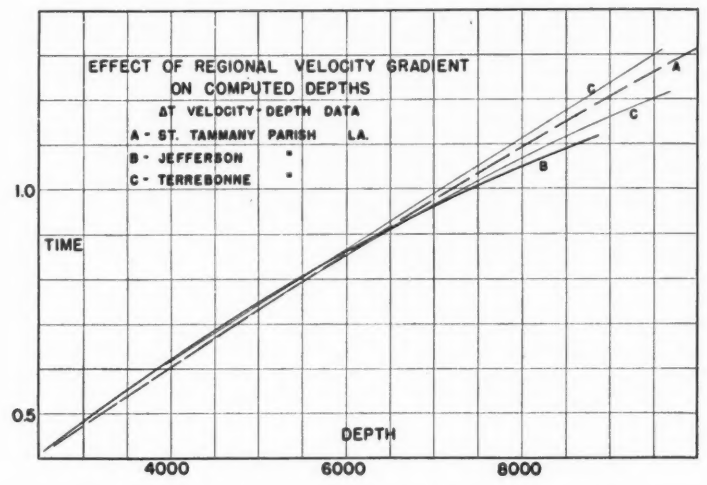

FIG. 9. Time-depth curves showing regional velocity gradient.

CONCLUSIONS

Observations of Δt data from routine seismograph records can be made to yield velocities whose probable errors are in the neighborhood of one per cent at prospecting depths. Disregarding the effects of systematic errors, which probably are not large, the probable errors for the velocities can be computed by means of the standard or average deviations of the observed Δt's.

ACKNOWLEDGMENTS

The author wishes to acknowledge his debt to K. E. Burg for having assembled most of the data used in this paper, and to Messrs. Ellis Shuler and Felix Leach and Mrs. Alvine Dearth for assistance in making the computations.

COMPARISON OF WELL SURVEY AND REFLECTION "TIME-DELTA TIME" VELOCITIES*

W. E. STEELE, JR.†

ABSTRACT

Empirical curves are given, comparing directly measured well survey velocities and velocities indirectly determined by means of reflection data. The observed relationships are discussed.

The calculation of subsurface dips from reflection data depends fundamentally upon the average velocity-depth relation existing where the reflection recording is done. The most accurate method commonly used in determining average velocity is to measure it directly by lowering a geophone in a well and recording the times from shots near the surface. However, in many cases, probably in most cases where dip shooting is done with the "leapfrog" method of alternating shot points and recorder positions, the velocity used is determined indirectly from accumulated reflection "time-delta time" data. Thus, the results obtained by these two different methods can be compared wherever a well which has been surveyed for velocity is located nearby, or included in an area covered by reflection shooting. Fig. 1 illustrates such comparisons for five different areas in the Texas and Louisiana Gulf Coast. It is seen that in all the areas the "t-Δt" velocities are lower than the well survey velocities, by amounts varying from 3 to 9 per cent.

The evaluation of the average velocity from the reflection data was accomplished by averaging large numbers of reflection picks from which an average t-Δt curve was drawn. Care was taken to use only records which had been recorded in opposite directions from the same shot or recorder position so that the average stepout would closely approximate that for flat reflecting beds. The average velocity $\overline{V}$ was calculated by substituting the values of reflection time and "stepout" in the equations

$$\overline{V} = \left(\frac{4AX}{2t\Delta t + \overline{\Delta t^2}} \right)^{1/2}, \tag{1}$$

or

$$\overline{V} = \left(\frac{2AX}{t_m \Delta t} \right)^{1/2}, \tag{2}$$

* Presented at the Annual Meeting, Houston, Texas, April 2, 1941.

† Independent Exploration Co., Houston, Texas.

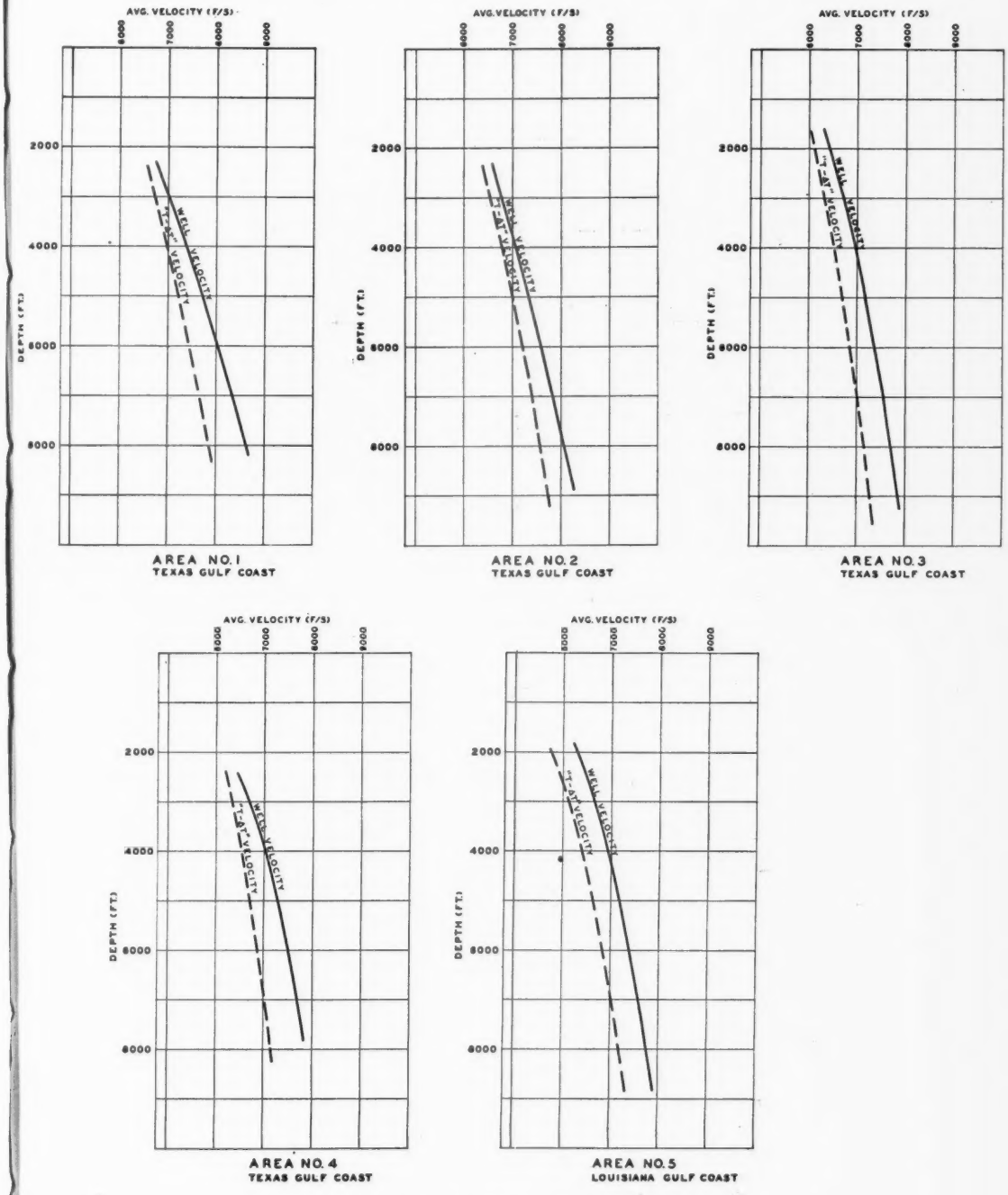

FIG. 1. Velocity-depth curves.

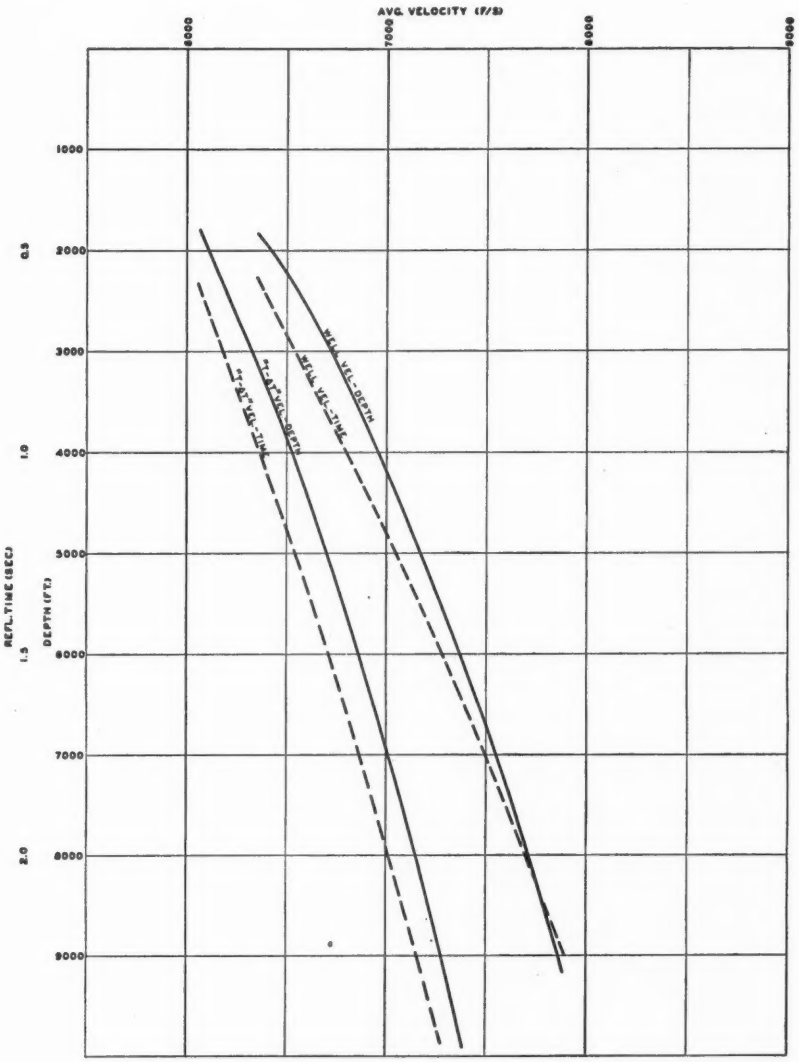

FIG. 2. Velocity curves for Area No. 3.

where $\overline{V}$ = Average vertical velocity.

t = First trace reflection travel time.

t_m = Mean travel time to first and last geophones.

Δt = "Stepout" time between first and last geophones.

A = Half the recorder spread.

X = Mean distance from shot to geophones.

In Fig. 2 the velocity-depth and velocity-time curves for Area No. 3 are shown on a larger scale. Here the t-Δt velocity is 4.8 per cent lower at 2,000 feet and 7.4 per cent lower at 9,000 feet.

The data upon which these plots are based are considered reliable, as large numbers of good reflections were used in arriving at the results.

Assuming that in the lower Gulf Coast the t-Δt velocities are invariably lower than the directly measured well velocities, and it appears that this assumption has good foundation, it follows that the reason or reasons for this relationship may be found in one or more of the following:

(1) Neglect of curved path in velocity calculations.

(2) "Picking" reflection travel time from cycle later than the phase change.

(3) Existence of multiple reflections or reverberations that affect the average t-Δt curves.

1. *Neglect of Curved Path*

It appears reasonable to assume that the effect of neglecting curved path can be discounted because several of the wells were surveyed for velocity with distances from shot to well of the same order of magnitude as half the mean reflection recording spreads used. The following example gives a measure of the error caused by neglect of curved path.

Example:

The true average vertical velocity as measured by a well survey in a certain area is shown in Fig. 2. Let it be assumed that reflection recording instruments are set up in this area, with distances of 2,000 feet and 3,000 feet from shot point to first and last geophones, respectively. Assume that the surface is flat and that at 5,000 feet subsurface lies a plane reflecting interface, parallel to sea level. At five thousand feet the plot shows the average velocity to be 7,160 feet per second.

It has been proved that if the instantaneous velocity varies linearly with the depth so that $V = Vo + CD$, then the reflection time for a sound ray is given by the equation

$$t = \frac{2}{C} \cosh^{-1} \left[1 + \frac{C^2 \left(\frac{X^2}{4} + D^2 \right)}{2V_o(V_o + CD)} \right]$$

where t = Reflection travel time.
C = Rate of velocity increase with depth.
X = Distance from shot point to geophone.
D = Depth of reflecting interface.
Vo = Velocity at surface of ground.

The linear depth equation closely approximating the well survey velocity of Fig. 2 is

$$V = 6{,}130 + 0.428D.$$

Thus the curved path t for the 2,000 foot geophone is calculated to be 1.4264 sec., and for the 3,000 foot trace 1.4601 sec. The Δt would then be 0.0337 sec. and the mean reflection time 1.4432 sec. Substituting this Δt and mean reflection time in the straight path equation

$$\overline{V} = \left(\frac{2AX}{t_m \Delta t}\right)^{1/2}$$

gives $\overline{V} = 7{,}169$ feet per sec.

Similar calculations made for depths of 3,000 feet and 8,000 feet are shown in the following table.

TABLE 1

Depth	$\overline{V}$	*Curved Path* t (2,000′)	*Curved Path* t (3,000′)	Δt	*Calculated* $\overline{V}$
3000	6720	0.93639	0.99294	0.0566	6766
5000	7160	1.42640	1.44325	0.0337	7169
8000	7700	2.08939	2.10916	0.0198	7755

From this example it appears that for flat reflecting surfaces the error caused by neglect of curved path is within the observational error.

2. *Effect of Picking Reflection Travel Time from Cycle Later than Phase Change*

Frequently the travel time and stepout of a reflection is picked on the strongest cycle following the phase change. Assuming that the Δt of this cycle is the same as that at the phase change, the effect can be studied from the equation

$$\overline{V}' = \left(\frac{t_m}{t_m'}\right)^{1/2} \overline{V}$$

where $\overline{V}$ = True velocity.
$\overline{V}'$ = Calculated velocity.
t_m = Mean reflection time at phase change.
t_m' = Mean reflection time other than at phase change.

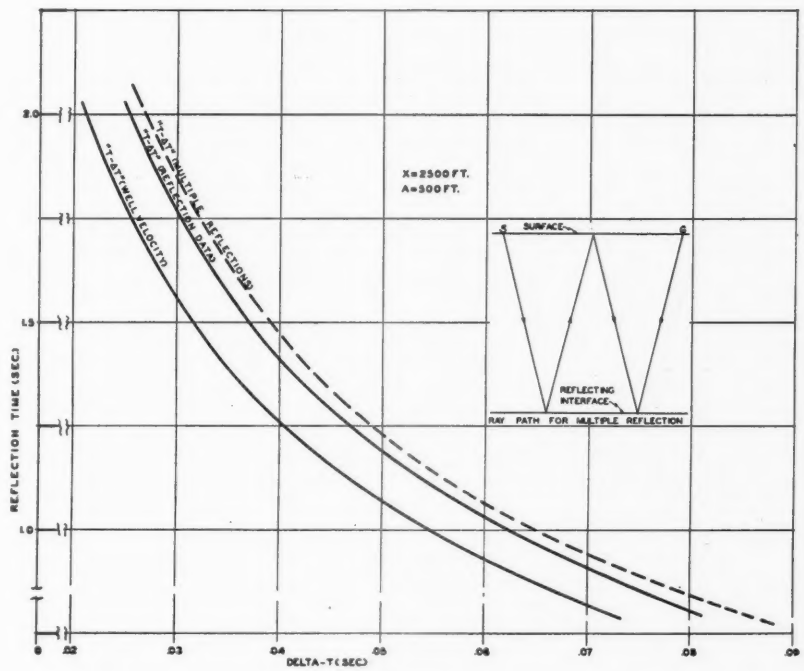

FIG. 3. "t-Δt" curves.

If the reflection is picked 0.03 second later than the phase change at a phase change time of 1.5 seconds

$$\overline{V}' = \left(\frac{1.5}{1.53}\right)^{1/2} \overline{V}$$

$$\overline{V}' = 0.9901 \overline{V}$$

or if

$$V = 7280 \text{ (Fig. 2)}$$

$$\overline{V}' = 7280 \text{ Feet/sec.}$$

Similar calculations were made for 1.0 sec. and 2.0 sec. and the results are tabulated as follows:

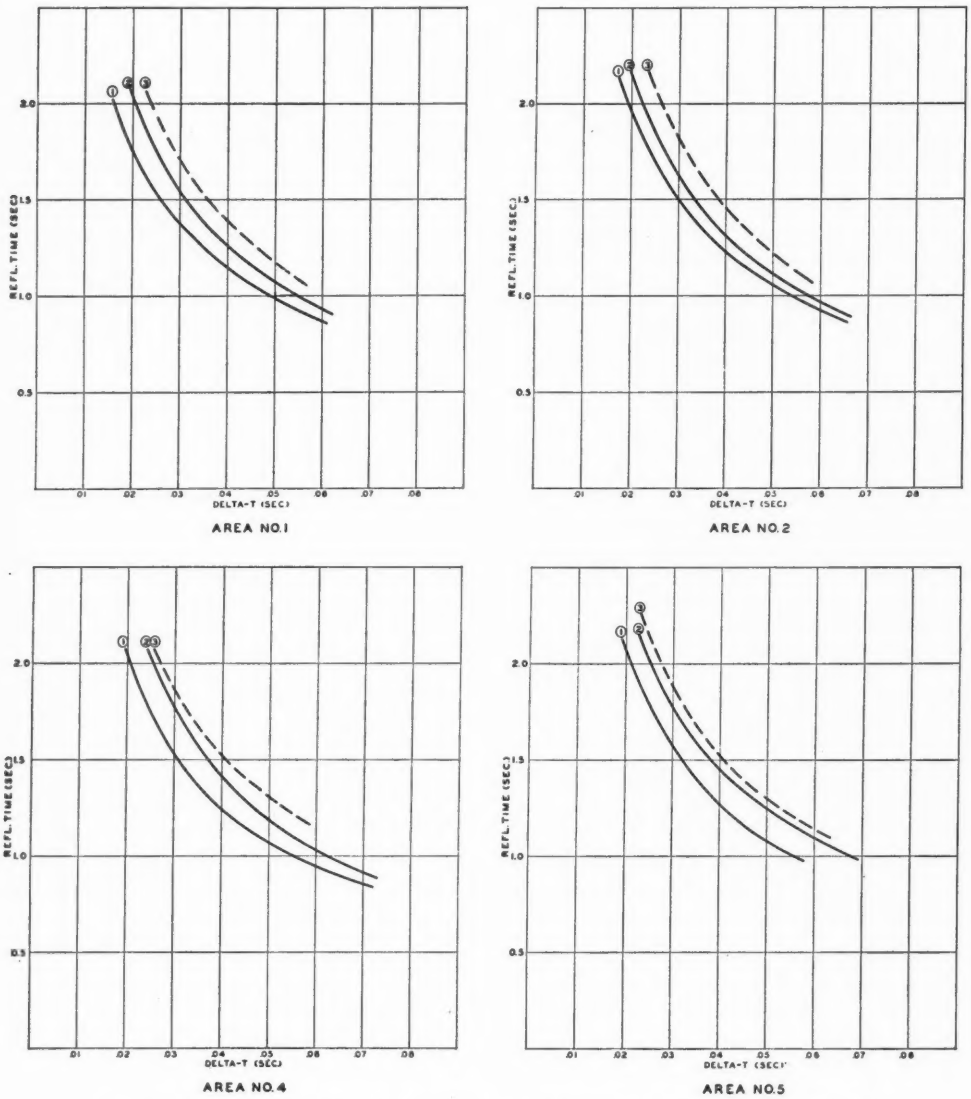

FIG 4. "*t*-Δt" curves. Curves 1—Well survey, 2—Reflection data, 3—Multiple reflection.

Mean Reflection Time	$\bar{V}$	*Average Vel.* $\bar{V}'$ *When t is .03 sec. late*
1.0	6800	6700
1.5	7280	7208
2.0	7690	7633

This shows that picking the travel times late lowers the calculated t-Δt velocities but not sufficiently to account for the observed differences.

3. *Effect of Multiple Reflections*

In Fig. 3 are plotted the observed t-Δt curve and that calculated for the well velocity in Area No. 3. If it be assumed that some of the sound rays follow paths similar to that shown in the diagram in Fig. 3, that is, from the shot to the reflecting interface, to the surface, back to the reflecting interface and then to the geophone at the surface, a t-Δt relation different from either of the other two would result. Such reflections could be called second order reverberations in that they travel to and from each reflecting interface twice. The dashed curve of Fig. 3 shows that t-Δt relation calculated for such conditions with the well survey velocity as a basis. Its nearness and similarity of shape to the observed t-Δt curve is striking. Similar curves showing these relations for each of the other Gulf Coast Areas are shown in Fig. 4. From these illustrations it appears likely that multiple reflections are recorded along with the direct reflections and considerably affect the velocity calculations based upon assembled reflection data.

NOTES ON REFRACTION PROSPECTING*

CHARLES HEWITT DIX†

ABSTRACT

This is a series of notes dealing with several aspects of the problem of the interpretation of refraction data. A method of dealing with unreversed refraction lines is outlined. The case of a high velocity section with an overburden having a linear increase of velocity with depth is discussed. We show how the accuracy of the relative depth calculations depends upon the difference between the upper and lower velocities at the interface in question. A method of computing tables for the linear velocity depth case is given which is well adapted to machine calculation. Finally, reflections at the ground-air interface and the formation of shadows by such obstacles as high or low velocity layers are considered.

INTRODUCTION

The following items include a number of contributions the writer has made to the science of interpretation of seismic exploration data during the past five years. In some cases they are labor-savers in the interpretive art. In others they give the interpreter a deeper insight into his data.

Many readers who have followed the trends of geophysical exploration may ask if it is not stepping backwards to consider the refraction seismic method at all. The answer is most emphatically no! The refraction method still has its place in many exploration programs. This is especially true in those cases where we must look for faults—a task for which the reflection method is generally poorly suited.

To make this statement more explicit, we should add that we do not refer to the type of fan-shooting used for the detection of Gulf Coast salt domes. We mean adequately reversed refraction profiles, measured with well designed, modern instruments.

The refraction method is superior to the reflection method for finding faults, for several reasons. The most serious objection to the reflection method is that reflections are seldom well defined and clear-cut in the neighborhood of a fault. Even gentle folding can produce the effects shown by faulting on poor reflection records. On the other hand the refraction line showing a fault by shooting in one direction, can be checked by reversal. The wave used in refraction prospecting is less disturbed by the zone of faulting than is the wave used in reflection prospecting, since the principal part of the reflected wave

* Presented at the Annual Meeting, Houston, Texas, April 2, 1941.

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must travel wholly within the disturbed zone. On the other hand, the "refracted" wave travels across the disturbed zone, traversing a greater proportion of its path through less disturbed sedimentary rocks.

Further difficulties arise in connection with the reflection method if too many reflections are present so that correlation must be carried on with continuous time ties. In such a case, even a large displacement would be likely to pass unnoticed on a reflection survey. However, unless the changes of physical properties of the section with depth are unusually small, velocity changes will be sufficient to show the fault if its vertical displacement is not too small.

REFRACTION PROFILES

Under this heading we shall discuss refraction profiles, reversed and unreversed. It was the author's good fortune to have many old refraction lines presented to him for re-interpretation. These lines covered a great variety of geological conditions. Likewise there was a great diversity in the quality of the data. In many cases the data were very incomplete and it was necessary to fill the gaps in the data by making various hypotheses. By such additions one reduces the definiteness of the conclusions. The writer's problem was to make the most of the incomplete data.

Methods available appeared to be excessively laborious. The use of complicated formulas arising out of Snell's Law makes numerical errors likely as well as difficult to find. Furthermore, Snell's Law presupposes a knowledge of the velocities which is not always available, especially in unreversed lines.

The problem then, is to simplify the calculations and to refer the *first stages* of calculation to *given data*. Furthermore, the first results should be of a *fixed nature* corresponding to the *fixed data*, so that adjustments corresponding to various secondary hypotheses do not involve complete recalculations.

SIMPLEST CASE

The simplest refraction problem occurs when we are concerned with just two layers of constant velocity. If the interface separating these two layers is a plane, the data that we receive by an unreversed refraction line can be partially interpreted without knowing the velocity of the lower medium. It is important to realize that only the apparent velocity due to the lower medium is given by the data. Such an

assembly of data is shown in Fig. 1, in which times, as ordinates, are plotted against corresponding distances as abscissas. In Fig. 1 the slope of the first part of the time-distance graph measures the velocity in the upper medium. The slope of the line on the right hand side of

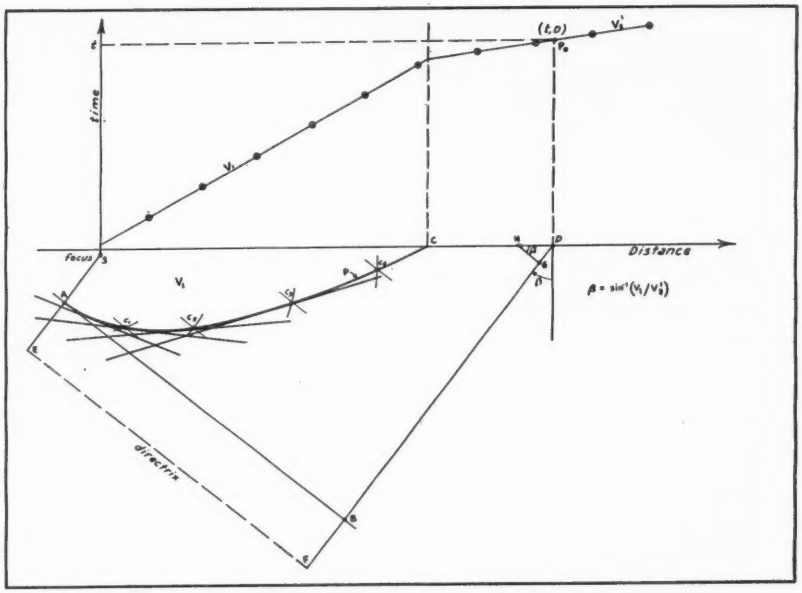

FIG. 1. Showing method of constructing parabola to which the refracting interface must be tangent if it is a plane and if the layers have constant velocity.

the time-distance graph measures the apparent velocity of the wave coming from the lower medium. Suppose we select a point on the part of this graph corresponding to the apparent velocity of the lower medium (call this point P_0). This corresponds to the distance D from the shot-point. Because we know the apparent velocity measured at D, and also the velocity of the upper medium, say V_1, we can determine the direction which a ray takes in coming up towards the point D from below. The angle which this direction makes with the vertical is denoted by $\beta = \sin^{-1}(V_1/V_2')$ where V_2' is the apparent velocity due to the wave from the lower medium. This ray direction, which is FBD in Fig. 1, can be drawn with the aid of a protractor.

Now it must be remembered that we know nothing regarding the velocity in the lower medium. This velocity can range between wide

limits. It must be higher than the velocity of the upper medium, and it must be less than +infinity. In case the velocity was +infinity (an obviously impossible case which would correspond to a perfectly incompressable medium), the location of the interface could be found easily. In fact, the interface shown on the figure as AB would be the interface in question. Because of the infinite velocity in the lower medium, the angle of incidence is o°, the travel time from A to B is zero and hence, $t=(\overline{SA}+\overline{BD})/V_1$. This then determines the extreme interface AB.

We should like to determine a curve to which all possible plane interfaces must be tangent in order to give the fixed data shown in Fig. 1 and will find very simply that this curve is a parabola which is very easy to construct. We proceed in the following manner: draw the line DF, where the distance, $\overline{DF}$, is equal to V_1, times the travel time to D denoted by t. At F, construct the perpendicular EF to DF, and use this line, EF, as the directrix for the parabola whose focus is at shot-point S. Now a parabola is a locus of points whose distances from a fixed point, or focus, are equal to their distances to a fixed line, which is the directrix. Thus, we have constructed a series of points A, C_1, C_2, C_3, and C_4. A is midway between S and E, and thus is at the same distance from the fixed point as from the fixed line. C_1 is at the same distance from the fixed point S as it is from the fixed line EF. Similar remarks apply to the other points on the parabola. The construction of these points is quite obvious. A series of lines is drawn parallel with the directrix EF. Setting the radius of the compass equal to the distance from one of these lines to the directrix, and placing the center of the compass at the point S strike a small arc cutting its corresponding line. This point of intersection determines a point on the parabola. Through a series of such points the parabola can then be drawn with the aid of a French curve. Various interfaces tangent to this parabola are shown in Fig. 1.

This parabola has some rather interesting properties, the essential discussion of which was contained in one of the writer's earlier papers.[1] It will be noted that the parabola cuts the datum plane surface at the point C, which is the critical distance at which the time distance curve takes a sudden turn. We may also observe that the ray drawn from S to the refracting-plane interface meets the plane at the place where it is tangent to the parabola.

[1] C. H. Dix, Journal of the Society of Petroleum Geophysicists, Vol. 6, No. 1, July, 1935.

The advantage of constructing this parbaola in refraction computing, in which we are dealing with layers of constant velocity, is fairly evident. The parabola depends only upon the data and upon the hypothesis that the interface is a plane. Thus the first steps in our computation involve no subsidiary hypotheses regarding the dip of the interface or the velocity of the second medium. Such a method of computing can be applied readily to unreversed refraction lines and to partially reversed refraction lines. The method is easily extendable to the case of several layers, by the well known process of successive removal of the upper layers. Such a process is evidently subject to inaccuracies due to the fact that the location of the "equivalent" shot-point at a buried interface is dependent, in each case, upon a knowledge of the dip of that interface. However, it can be established by trial, that a considerable error in the dip of, say, the first interface, does not make a serious error in the location of the parabola determining the second interface.

It may be mentioned that other inaccuracies become very much more important as the number of layers increases. The velocities are usually confined between 1,600 meters per second and 6,000 meters per second, so, if the velocity is non-decreasing with increasing depth, the larger the number of layers, the smaller will be the size of the velocity difference between layers. As the velocity difference between layers becomes smaller and smaller, the errors introduced by inaccuracies in measuring the data become more and more serious. The writer has applied this method to as many as four layers, but believes that the accuracy of the location of each individual interface is not satisfactory in such a case.

The interpreter will find that the parabola is very easy to draw, because the two points A and C are already determined on the parabola. For many purposes, one or two additional points will be sufficient.

The writer has found that useful information can be obtained in an area where scattered unreversed profiles have been taken by the following expedient. Draw the parabolas corresponding to the refraction profiles on long strips of graph paper, to a scale of, 1:20,000. Place these parabolas in their proper position on a map to the same scale, so that the plane of the parabola is vertical over its corresponding refraction line. Then the interface in question must be tangent to all of these parabolas in the area. Irregularities, which are the chief interest of the geophysicist, will show up by failure of a single plane to be

tangent to all of these parabolas in the area. Some care should be exercised in this matter, because the failure may come about because of the fact that the interface is not a plane directly beneath some of the refraction lines in question. It has been found that this lack of planeness can be definitely established in certain cases where the refraction lines are close enough together, even though the lines are unreversed.

The writer does not recommend the use of unreversed refraction lines as a regular procedure. By the addition of a reversal, which may double the cost, much more than double the amount of information is obtained, because we obtain in this way a measure of the actual velocity in the lower layer, and so a measure of the dip in that layer. Furthermore a change of dip is much easier to detect on a reversed refraction line. The special utility of the parabola method of computing is most evident in case the data is old and perhaps obtained without a complete understanding of the possibilities of the refraction method.

CIRCULAR RAYS AND SPHERICAL WAVE FRONTS

In many cases the geological section is such that the assumption of a small number of layers of constant velocity is very far from the truth. If we were to measure the velocity as it depends upon the depth, it would be found that this velocity is quite variable. It varies rather rapidly from depth to depth with rapid changes of lithology and of course it is quite impossible in either refraction or reflection work to take into account all the details of this velocity variation. However, we can substitute for this velocity distribution one which is very much simpler and which will allow us to carry on calculations with considerable ease without introducing errors much greater than the observational errors of the data.

We shall consider in this section velocity distributions which are such that the rate of increase of the velocity with the depth is constant. In such a case, as is well known, the rays are circular arcs, and the wave fronts are spherical surfaces always perpendicular to the circular arc rays. If we consider a plane section, say any plane drawn through the shot-point, the intersections of the spherical wave front surfaces on this plane will be circles. In order to simplify the problem, we shall assume that the velocity increases with vertical depth, and furthermore that the plane sections are vertical plane sections. In case these assumptions appear not to correspond to the facts, care should be taken to remember that a certain amount of adjustment

will be required. The main properties of the ray circles are derived in Eve and Keyes' "Applied Geophysics," at the end of the book. For us, the important property of the ray circles is that all their centers are located on a single plane or, in our case, on a single line above the datum surface.

Let the velocity distribution be given by the equation

$$V = V_0(1 + kH), \tag{1}$$

where H is the depth below the datum.

Observe that when H is zero, we are at the datum level. The value of V at the datum level is V_0. The rate of change of velocity with depth is kV_0. Thus k is a "specific" rate of increase of velocity with depth. The height of the line of centers above the datum line is found to be $1/k$. This is established in a number of places, including Eve and Keyes, and so will not be further derived here.

The problem of determining the values of k and V_0 from the refraction data is not difficult. It is easy to construct time-distance graphs corresponding to a series of values of k for a series of values of V_0. These can be constructed on transparent material and placed over the measured data, and compared thereto. The writer believes that this process is somewhat superior to the process of determining V_0 from the early part of the time-distance curve, and of determining k from the latter part of the time-distance data. This latter process places too much emphasis upon the velocity of the very shallow material. There is an approximate analogy to the following process. Suppose the velocity in a well is determined as a function of the depth of the well from some datum surface on down at intervals of say 100 ft. If the velocities so determined are plotted against the depth a rather scattered series of points will be found. The "best" straight-line fit to the series of points can be determined either by requiring that the straight line go through the point corresponding to the datum velocity, or one can determine the straight line without requiring that it go through any single point. Evidently, if the data are disturbed in the shallower points, the precision of fit will be seriously reduced by forcing the line through the observed datum velocity.

Now suppose that these time-distance curves have been applied to the data and the values of k and V_0 have been determined. Then, knowing k, the line of centers of the ray circles a distance of $1/k$ above the datum line can be drawn. This has been done in Fig. 2, where ll' is the line of centers of the ray circles, and LL' is the datum line. Sup-

pose we consider the preliminary problem of determining the ray arc between the fixed points A and B as shown in Fig. 2. We have only to construct a circle passing through A and B having its center on the line of centers ll'. The center of this ray circle will also be on the

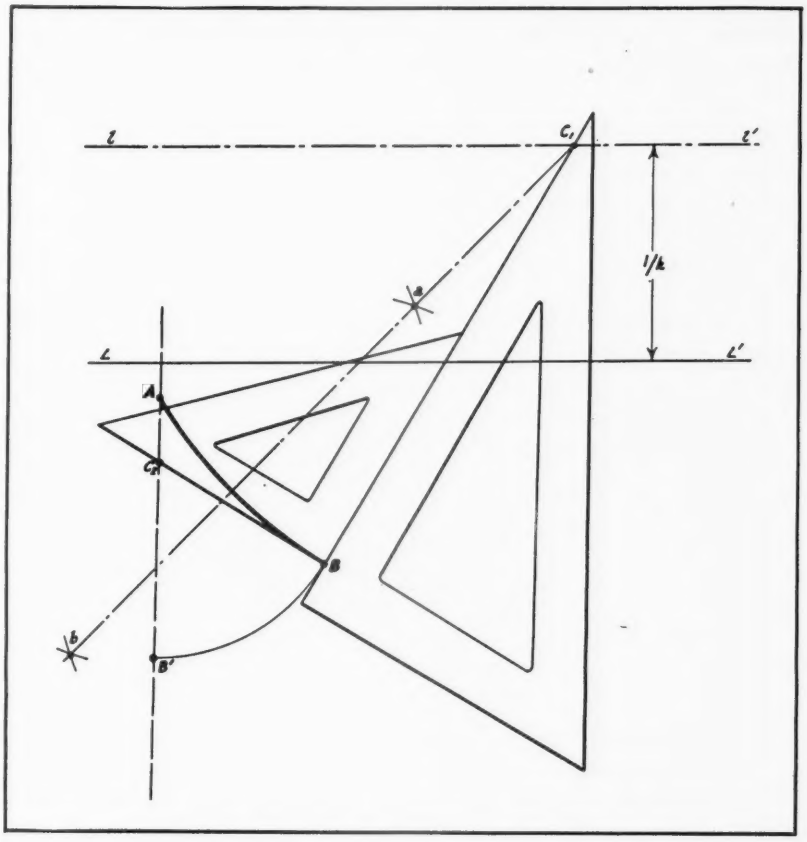

FIG. 2. Showing method of constructing ray arc between two given points (A and B) and determination of travel time.

perpendicular bisector of the segment joining A and B. This is easily constructed by placing the compass center at A and swinging two arcs, and then, with the same radius, swinging arcs at B cutting the same arcs at the points a and b. These two points, a and b, determine the perpendicular bisector of the segment AB. This perpendicular

bisector intersects the line of centers ll' at the point C_1 which point is the center of the ray circle through A and B.

The wave front circle can easily be constructed as shown in Fig. 2, by placing the larger triangle with the straight edge on the points C_1 and B and placing the smaller triangle along this same edge with its right angle corner at the point B. This smaller triangle has an edge which intersects the vertical line through A at the point C_2. This point C_2 is the center of the wave front circle through B when A is the source

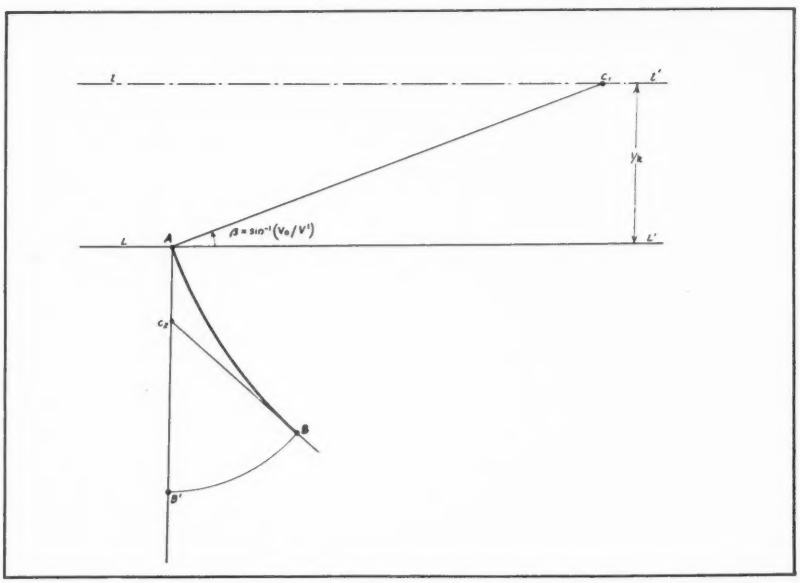

FIG. 3. Showing construction of ray arc when the angle of emergence β is given.

of the wave. The travel time from A along the ray to B is then the same as the travel time from A to B'.

This is a very important construction because of the fact that the formula for the travel time from A to B is a very difficult one to apply, whereas the formula for the travel time from A to B' is very simple to apply. Thus we calculate the travel time from A to B by calculating the travel time from A to B'.

In refraction problems, the two fixed points as shown in Fig. 2 are not usually given. Rather we are given one fixed point at the datum level together with the direction that the ray takes in approaching

point A. This direction is determined by the apparent velocity of the wave coming up to A and by the datum velocity V_0. The situation is illustrated in Fig. 3, in which the angle which the ray makes with the vertical is denoted by β and is given by the simple inverse sine formula

$$\beta = \sin^{-1}(V_0/V').$$

In fact, the line shown in Fig. 3 and denoted by AC_1 is parallel with the wave front at the point A as this wave front comes up towards A. The perpendicular to this line (AC_1) at A is tangent to the ray path BA at the point A, and this perpendicular makes with the vertical AB', the angle β. It is important to observe that *the point* C_1 *is constructed from the data alone* and is determined from the knowledge of k, V_0 and the apparent velocity V'. Our problem then, will be to determine the travel time from A to *any* selected point B along this ray. Now the travel time from B to A is the same as the travel time from A to B. Hence it may be imagined that the point A is a source of spherical waves merely for the purpose of determining travel time to B. We then construct the wave front circle BB' and determine the travel time from A to B' which is equal to the travel time from B to A and was to be determined. The formula for the vertical travel time from A to B' is very easily computed, and is

$$t = (1/kV_0)\log_e(1 + kH). \qquad (2)$$

Tables of values of t and H can be constructed from this formula by a method to be described in a later section.

Fig. 4 shows a complete reversed refraction line including the final steps in the calculation. The values of k and V_0 have been determined by placing standard curves over the data. Thus the line of centers ll' is determined. Furthermore the apparent velocities V' and V'' of the high velocity layer have been determined from the data as shown. From V' and V'' the two angles β' and β'' are determined, as shown in the figure. These angles suffice to give the determination of the centers of the two ray circles corresponding to the rays coming up to the points A_1 and A_2. These ray circles are drawn. It is then assumed that the velocity of the high velocity layer is approximately the average of the two apparent velocities V' and V''. The approximation made in this case is very close to the correct value that would be found by a more detailed analysis, and is sufficiently accurate because of the fact that the observational errors in the determination of V' and V'' are usually larger than the error made in taking the simple average.

The author's original method of determining the depth of the high velocity layer involved the use of a formula which, it is found, can be discarded. Rather than determine accurately the depth to the high velocity layer, the writer merely estimates the depth as closely as

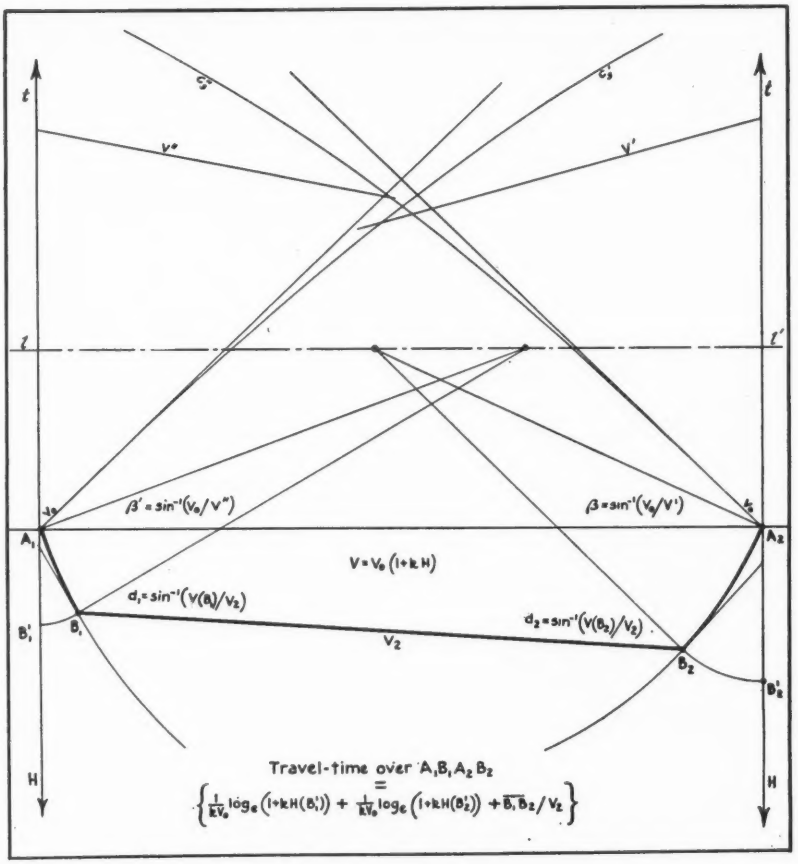

FIG. 4. Showing arrangement for computing reversed refraction line.

possible, and draws in an interface, say B_1, B_2. Not only is the depth estimated, but also the dip. Then the travel time over the path $A_1B_1B_2A_2$ is computed and compared with the measured time from A_1 to A_2. If the computed time is larger than the measured time, the high velocity interface is evidently too deep and likewise if the computed time is less than the measured time, the high velocity interface is evidently too shallow.

The problem then is to make an accurate adjustment of depth so as to bring the computed interface into the correct average position in as few steps as possible. This has been done with the aid of the depth adjustment curves which are shown in Fig. 5.

In Fig. 5, the abscissa represents the velocity on the lower side of the interface, that is, V_2 ,and the ordinate represents the velocity on the upper side of the interface, which is computed by the use of the formula which relates the velocity to the depth. Thus these two veloci-

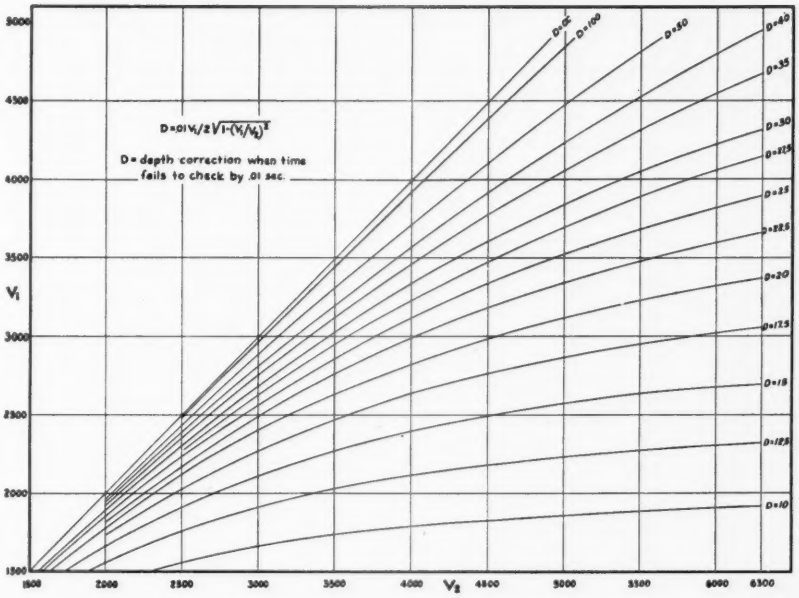

FIG. 5. Series of curves each corresponding to a fixed amount of depth adjustment per 0.01 sec. time adjustment.

ties locate a single point on the graph of Fig. 5. Fig. 5 represents a series of curves, each corresponding to a fixed amount of depth adjustment per 0.01 of a second time adjustment. It will be found that the adjustments range in practice between about 15 meters and 40 meters per 0.01 second of time adjustment. It is seen from the adjustment curves that a large velocity contrast corresponds to the small depth adjustment. In fact the series of curves illustrates, in a very lucid manner, how the accuracy of the depth determinations decreases as the velocity contrast decreases. The method of calculating the points for the curves of Fig. 5 is quite simple, and will not be considered here.

Having determined the amount of depth adjustment required by comparing the computed travel time over the estimated path with the measured travel time, and by using the curves of Fig. 5, the corresponding adjustment to the interface shown in Fig. 4 is made. It will usually be found that two such adjustments will be ample to bring the difference between the computed time and the observed time to within 0.005 second.

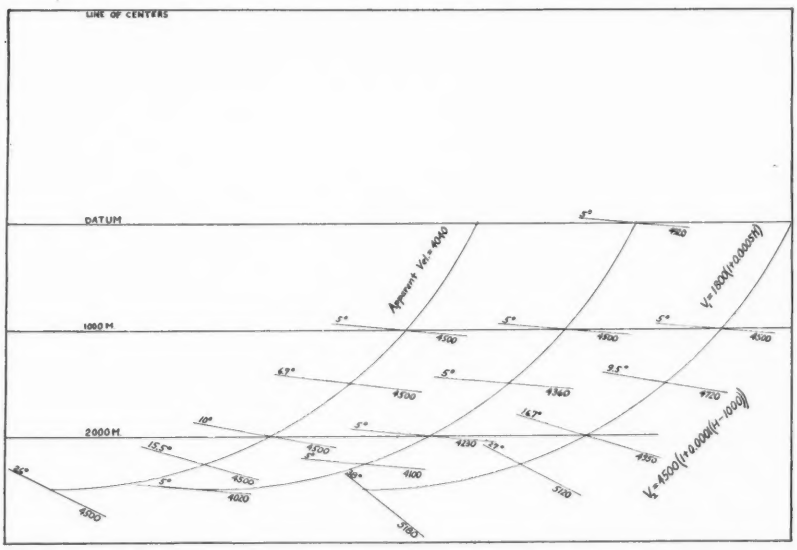

FIG. 6. Dip variation with depth.

The process of making the estimates becomes more and more accurate as experience in making these estimates is gained, and eventually the interpreter will find that many of his first estimates fall within the range of probable error of observation of the observed time. In any case a single adjustment will usually bring the interface to its correct position. The same remarks apply to estimates of the dip of the interface. The dip angle is related to the angles d_1 and d_2, which the wave fronts make with the refracting interface. These angles are shown in Fig. 4. The expression for these angles in terms of the inverse sines can be used for a quick slide-rule computation of these angles to check the dip.

When dips become greater than 5° or 10°, some care should be

exercised in the interpretation because of the fact that a plane interface over a layer of constant velocity V_2 will not produce straight line time-distance graphs giving accurate determination of the apparent velocities V' and V''. In other words, if the time-distance graphs are straight lines, the interface can not be plane, unless the two apparent velocities are equal, i.e., unless there is no dip at all.

The situation with respect to the dip variation with depth is shown in Fig. 6. Three separate ray circles are shown, each corresponding to an apparent velocity of 4,040 meters per second. On the left hand ray circle, it is assumed that the interface is above a layer of constant velocity of 4,500 meters per second. At a depth of 1,000 meters, this corresponds to a dip of 5° as shown by the upper segment of the series. At depths of 1,500, 2,000, 2,350, and 2,500 meters, the dips are respectively 6.7°, 10°, 15.5°, and 26°. On the middle ray circle, is shown a situation which occurs when the dip angle is held fixed, and the velocity of the high velocity layer is allowed to vary. Then the velocity corresponding to zero depth is 4,920 meters per second. At 1,000, 1,500, 2,000, and 2,350, and at tangency, these velocities are respectively 4,500, 4,360, 4,230, 4,100, and 4,020 meters per second. This situation corresponding to a fixed dip, or, in other words, to a plane interface, can usually be pretty well ruled out, because it involves the consequence that the velocity in the high velocity layer decreases as the depth increases. This situation is very improbable, since wherever the velocity can be measured in a medium of otherwise fixed lithology, this velocity shows a tendency to increase with increasing depth. The ray circle on the right hand side of Fig. 6 shows what happens when the velocity in the high velocity layer is allowed to increase with depth. The increase is taken, in this case, to be a linear increase with depth, and here the dip at 2,500 meters increases to 38°.

In a few cases in actual practice, the writer has had occasion to take into account this change of dip with depth. In fact, in a few cases, this variation has been required in order to give a continuous tie on the high velocity interface from one refraction line to another.

The method outlined is especially suitable when the over-burden above the high velocity layer is a section made up principally of tertiary sands and shales of varying composition. When thick sections of an intermediate high-velocity material, such as salt occur, as for instance the salt of Yoakum County, West Texas, the method is not suitable for determining the top of a deeper high velocity layer. Ob-

viously the reason is that the actual velocity distribution departs greatly from a linear distribution.

This method of computing is simple and can be applied rapidly enough so that the interpreter can follow the field work currently and have a preliminary interpretation ready only about half a setup behind the progress of the shooting crew. This is a great advantage, because it allows the party chief efficiently to direct the work of securing the data as and where needed. Former practice of securing the data and interpreting it days or even weeks or months later, made it impossible to direct the field work in an intelligent manner. Field work in that case had to be carried out in a routine manner, and many irregularities had to be treated as best they could by the interpreter after the field work had been closed. And since in most cases these irregularities constituted the portion of greatest economic importance, the efficiency of the survey was greatly decreased.

CONSTRUCTION OF TABLES

The construction of tables giving a series of depths in terms of a series of times, can be greatly facilitated by the following procedure. We can solve Eq. (2) for the depth H and get

$$H = (e^{kV_0 t} - 1)/k. \tag{3}$$

We may then substitute the various values of t in the exponential and with a good table of exponentials easily calculate the values of H. Another arrangement which lends itself more readily to computation on a computing machine, is to derive from Eq. (3) a socalled recursion formula. In order to illustrate the method we shall calculate a small table using time intervals of .1 of a second. Denoting by $H(0)$ the depth when t is zero, by $H(.1)$ the depth when t is .1 of a second, and by $H(.n)$ the depth when t is $.n$ of a second. In Eq. (4) we see a series of values of depth for $.(n+1)$, $.n$, and $.(n-1)$ seconds.

$$\begin{aligned} H[.(n+1)] &= (e^{kV_{0}\cdot(n+1)} - 1)/k \\ H(.n) &= (e^{kV_{0}\cdot n} - 1)/k \\ H[.(n-1)] &= (e^{kV_{0}\cdot(n-1)} - 1)/k. \end{aligned} \tag{4}$$

Taking the difference of the first two and also of the last two equations, we see that the two differences are related, as shown in

$$\{H[.(n+1)] - H(.n)\} = \exp(kV_{0}\cdot 1)\{H(.n) - H[.(n-1)]\}. \tag{5}$$

Equation (5) is our recursion formula, which is used to compute the table. If we take $n=1$, the Eq. (5) becomes

$$H(.2) - H(.1) = \exp(kV_{0.1})\{H(.1) - H(0)\} = \exp(kV_{0.1})H(.1). \quad (6)$$

So in order to start the calculation we must first calculate a common exponential factor, and also calculate the depth corresponding to the time of 0.1 of a second. We then set the exponential factor in the calculating machine and multiply it by the depth corresponding to 0.1 of a second. This gives the value of the difference of depth shown on the left hand side of Eq. (6). This difference is then multiplied into the common exponential factor (which is left set in the machine), and the next difference in depth is obtained. By a continuation of this process, we calculate a series of differences of depths corresponding to .1 second intervals in time. By adding the column of differences we get the successive values of the depths.

It will be observed that this procedure will lead to a serious error, if there is an appreciable error in the calculation of the common exponential factor or in the first depth. It is, therefore, important to calculate these first two quantities with sufficient accuracy to give the required accuracy by the time the end of the table is reached. Usually the common exponential factor has an exponent which is quite small, hence the series for the exponential, which is

$$\exp(x) = 1 + x + x^2/2! + x^3/3! + \cdots + x^m/m! + \cdots \quad (7)$$

converges very rapidly, so that only a few terms in this series are required to give considerable accuracy in the determination of the exponential.

It may be observed that the above process can also be used in determining a table of reflection times. In such a case we can use the recursion formula (Eq. (5)) except that in this case the time is one-half the reflection time. This will introduce the factor $\frac{1}{2}$ into each of the exponential terms whenever the time refers to the reflection time.

REFLECTION ON THE TOPOGRAPHIC SURFACE

In earthquake seismology, the compressional wave travelling directly from the focus to the observation station is specified as a *P*-wave. The wave once reflected at the topographic surface is called the *PP*-wave. The situation is illustrated in Fig. 7, in which the single circular arc from *a* to *c* is the ray for the *P*-wave from *a* to *c*, and the two arcs, *ab* and *bc* are the rays for the *PP*-wave from *a* to *c*, once reflected at *b*.

The travel time for the *P*-wave is shown at *P* on the graph. The time for the *PP*-wave is shown at *PP* on the graph. Notice that the time *PP* is twice the time *E*, for the *P*-wave to travel to *b*.

From the point of view of geometrical optics, a shadow zone should be formed for the *P*-wave when there is an obstacle as shown in Fig. 7.

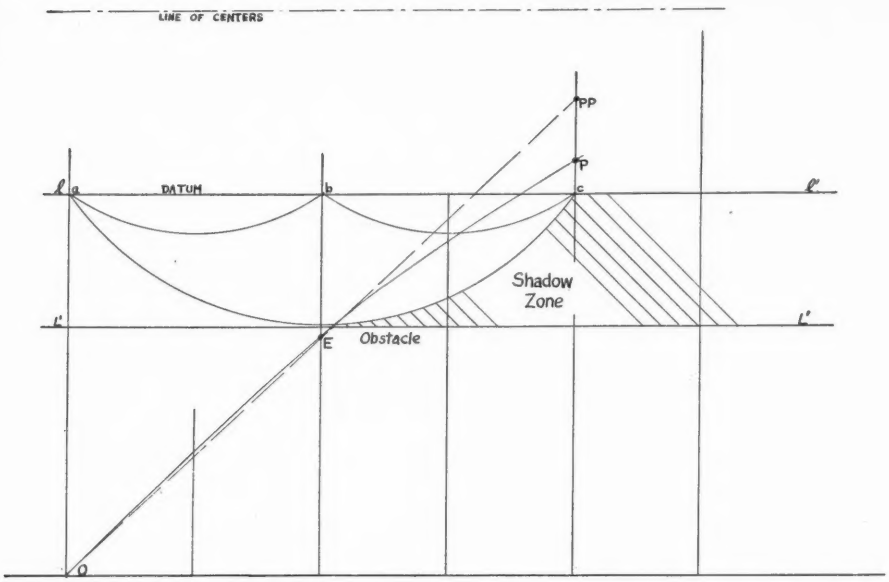

FIG. 7. Formation of shadow zone for *P*-wave.

However, the wave length will be about one-tenth of the total thickness of the over-burden between the obstacle *LL'* and the topographic surface *ll'*. Hence diffraction effects may be expected to be strong and a sharp shadow will not occur. However, a weakening effect should occur, and the *PP*-wave rapidly should become very much larger in relation to the *P*-wave as we pass farther and farther into the shadow.

Fig. 8 shows a set of traces of seismographs taken from a single shot-point. A linear velocity-depth relationship fits the *P*-wave's travel times very well. On the trace taken at 15,000 feet, the *PP'*-wave is shown following the *P*-wave. The *PP*-wave has a time 2.4 sec. at 15,000 feet, which would correspond to 1.2 sec. for the *P*-wave at 7,500 feet. Notice that the *P*-wave comes at about 1.2 sec. at 7,600 feet. The relative amplitudes of *P*- and *PP*-waves are difficult to find

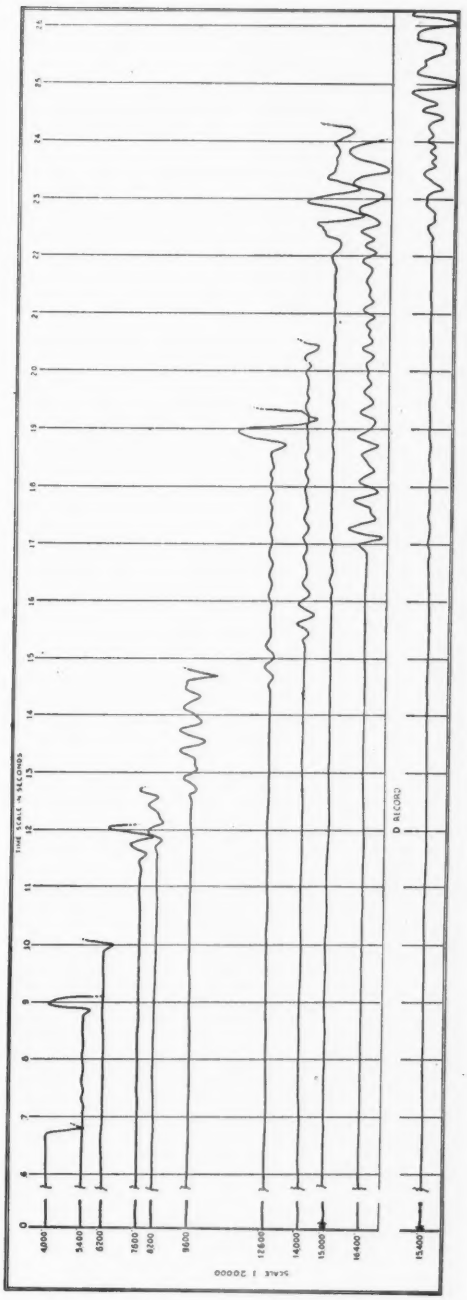

FIG. 8. A set of refraction seismograph traces taken from a single shot-point.

because the *PP*-wave can not be measured. However, on the *D* record below, the *PP*-amplitude is about 2.2 times the *P*-amplitude. The *D* record is over the same section but shot in the opposite direction. The shadow's edge is at 14,000 ± 1,500 feet. A record taken at 20,000 feet should have shown considerable extinction of the *P*-wave in comparison with the *PP*-wave.

Another effect is likely to enter due to the fact that the section is stratified and the velocity is not strictly a linear function of depth. This may contribute a complicated multiplicity to the phases composing the *P*-wave. That is, the *P*-wave itself is a combination of components introduced by velocity irregularities such as are almost certain to occur. However, since the actual velocity distribution is fairly well approximated by a linear distribution of velocity with depth, these various components all arrive in a group to form the *P*-wave seen for instance, on the trace for 15,000 feet.

In several cases the writer has observed that the velocity distribution in the section was such as to make the wave $P+PP+PPP+\cdots+P^{(n)}+\cdots$ predominant at the longer distances with the first terms of the series rapidly diminishing with increasing distance from the source. This has been especially marked when the sedimentary section shows a quite uniform lithology and when its thickness is somewhat restricted (say less than 3,000 feet to igneous basement). The restriction as to thickness is probably due to the fact that a shadow effect was never reached on a thick section because long enough refraction lines were never shot.

ACKNOWLEDGMENTS

The writer wishes to thank Mr. W. W. Hardy and the Socony Vacuum Oil Company for permission to publish this paper.

CURRENT PENETRATION IN DIRECT CURRENT PROSPECTING*

M. MUSKAT† AND H. H. EVINGER†

ABSTRACT

Using previously developed methods for calculating the potential about an electrode on the surface of a stratified earth, the fractions of total current, due to two current electrodes, flowing in a given layer and across the vertical plane mid-way between the electrodes, are calculated. The method employed is applicable in principle to any number of layers, and numerical values are given for two-layer and three-layer earths. Curves are given showing the variation of these current fractions with electrode spacing for the following types of conductivity stratification: two-layer earths with complete range of conductivity ratios; three-layer earths which are homogeneous except for the middle layer, the latter having thicknesses equal to or twice that of the top stratum; and three-layer earths having either infinitely conducting or resisting bottom layers and with middle stratum thicknesses equal to, twice, or three times that of the uppermost stratum.

It is virtually an axiomatic principle in geophysical prospecting that the greater the separation between the points of measurement on the surface the greater will be the subsurface depths which materially affect the relative values of the surface data. The spacing of gravity and magnetic stations is to a large extent determined by this rule, when considered in its simplest form. And when extended to cases where the reaction of the earth to an artificially excited impulse is measured, the same principle also applies broadly both to seismic and electrical prospecting.

In direct current electrical prospecting there are two kinds of spacing involved: (1) the spacing between the current electrodes, and (2) that between the potential measuring electrodes. If a fixed relationship be maintained between these two kinds of spacing such as is done, for example, in the Gish-Rooney method, then it is generally considered that an increase in spread of the composite electrode system will permit greater effective current penetrations than will a close spacing. On the other hand, if one pair of electrodes be kept fixed and the separation between the others varied, then again the deeper regions come into greater prominence as the separation is increased. These are generally accepted and well known observations in the field of electrical prospecting. Theoretically they derive their basis from the demonstrable proposition that the potential distribution

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about any single electrode asymptotically approaches that appropriate to the resistivity of the deepest layers as the distance between the current and potential electrode is indefinitely increased. When this specific result is applied to a composite potential and current electrode system by means of the principle of superposition, the more practical consequences previously stated follow at once.

One actually measures in direct current prospecting methods the apparent resistivity of the ground, which is essentially the potential difference between the two potential electrodes multiplied by an appropriate factor. Many theoretical curves have been calculated for the purpose of providing ideal resistivity curves corresponding to specific types of subsurface stratification. By comparison of the observed data with the calculated curves inferences may be drawn as to the probable actual conductivity stratification in the region being studied. Although additional information is frequently derived by further manipulation of the observed data, the essential principle of D. C. prospecting interpretation still remains that of comparison of observed and calculated data. In many instances this is attended with considerable success in predicting the presence of, and estimating occasionally even the approximate depth of, specific strata of particular interest. In other cases, however, the failure of the data to show sharp and critical characteristics has made it difficult, if not impossible, to deduce from them unambiguous conclusions.

Because of its very nature an apparent resistivity measured at the surface must necessarily represent the resultant of a complex average process applied to all the conducting beds underneath the surface. It is to be expected therefore that the surface data will in general be fairly smooth and continuous excepting, of course, for unavoidable experimental errors or surface resistivity variations. In fact if the surface layer resistivity is kept fixed, it is inconceivable that sharp discontinuities would result from such a diffuse system as the current distribution through a semi-infinite half space, to which the earth is essentially equivalent for the purpose of D. C. electrical prospecting. On the other hand, whatever sharpness the variations in the surface data possess will undoubtedly depend upon the particular conditions of the measurements. These include both the character of the subsurface stratification and the geometrical arrangement of the surface electrodes. In particular, it may be anticipated that the extent to which a specific conducting stratum will affect the surface resistivity data will depend directly upon the fraction of the total current flowing through

the earth which passes through the particular stratum. To bring out the presence of such an individual stratum the surface electrodes must therefore be so arranged that an appreciable part of the current will pass through it.

While this type of consideration has been tacitly recognized in most of the electrical prospecting literature, it has not been heretofore analyzed in detail,[1] excepting only for the rather trivial case of the homogeneous earth. For such a system it has been frequently stated in the literature that half the current introduced into the earth will pass entirely above a plane whose depth is equal to half the current electrode spacing. And a complete curve for the variation of the current penetration in a homogeneous earth with the depth has been published by Weaver.[2] Here we shall treat the more general and practical systems of stratified earths. We shall calculate the fraction of the total current introduced into the earth which passes through each of the several layers of the composite stratified earth as a function of the electrode spacing and of the conductivities of the various layers. Specific results will be presented for the two-layer and three-layer earths. The generality of the method, however, will permit direct extension to earth systems composed of still more layers, although the actual computations will, of course, be somewhat more laborious. Of course, no pretense is made that the results of this study can be used directly in electrical prospecting interpretation. Rather, they are presented only from the point of view of aiding in visualizing the nature of the current distributions in stratified electrical conducting media.

GENERAL THEORY

For the purpose of the present calculations it will be unnecessary to consider the positions and distribution of the potential electrodes. We shall be directly concerned only with the current electrodes. These we shall suppose to be placed symmetrically along the x axis at a spacing a from the origin, set on the surface as indicated in Fig. 1. The specific object of the calculations will then be the determination of the fraction of the input current flowing across a vertical section lying be-

[1] H. M. Evjen, GEOPHYSICS, *3*, 78 (1938), has indeed made a detailed analysis of the problem of the "weighting" of various subsurface strata in their effect on the apparent resistivity. His point of view, however, was based on the idea of relating the depth factors to the image systems corresponding to the D. C. potential distribution, so as to aid directly in depth estimates.

[2] W. Weaver, "Geophysical Prospecting," A.I.M.E., p. 68 (1929).

tween the depths z_1 and z_2. As this current will obviously vary with the position of the vertical plane for which the calculations are made, we shall choose that bisecting the two current electrodes, that is, the y, z plane. This is also the plane referred to in the calculations previously mentioned for the case of the homogeneous earth.

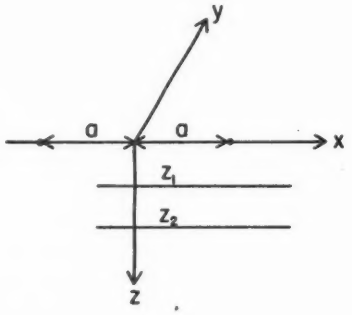

FIG. 1

If $\Phi(x, y, z)$ be the potential distribution in the stratum of interest, the current density across the y, z plane will clearly be:

$$i = -\sigma \left.\frac{\partial \Phi}{\partial x}\right|_{x=0}, \tag{1}$$

where σ is the conductivity in this stratum. The total current passing through the stratum will then be:

$$\int_{z_1}^{z_2}\int_{-\infty}^{+\infty} i\, dz dy = -\sigma \int_{z_1}^{z_2}\int_{-\infty}^{+\infty} \left.\frac{\partial \Phi}{\partial x}\right|_{x=0} dz dy. \tag{2}$$

When this is divided by the total current introduced at the input electrode, which we may denote by I, we shall then obtain the fraction of the total current passing through the stratum of interest. Using for this fraction the symbol f, we shall then have finally:

$$f = -\frac{\sigma}{I}\int_{z_1}^{z_2}\int_{-\infty}^{+\infty} \left.\frac{\partial \Phi}{\partial x}\right|_{x=0} dz dy. \tag{3}$$

Even without specifying in detail the geometric system, we can carry through the procedure indicated by Eq. (3) up to a final stage requiring only a single quadrature. For this purpose we notice first of all that because of the principle of superposition the resultant potential Φ can always be expressed simply as the sum of the potentials

due to the input electrode at $x=-a$ and the output electrode at $x=+a$. Each of these will moreover be axially symmetrical about the corresponding electrodes. In fact, as has been shown quite generally in a previous paper[3] these potential distributions may be expressed in the form:

$$\Phi = \int_0^\infty J_0(\alpha\sqrt{(x+a)^2+y^2})F(\alpha, z)d\alpha, \tag{4}$$

where F is a function depending upon the stratification of the subsurface and upon the conductivity distribution, and moreover possesses the property that the resultant integral has a simple pole at the electrode, that is, at $x=-a$, $y=z=0$. For the composite system in which we have the input electrode at $x=-a$ and output electrode at $x=+a$, we shall therefore have as the resultant potential distribution:

$$\Phi = \int_0^\infty F(\alpha, z)\left[J_0(\alpha\sqrt{(x+a)^2+y^2}) - J_0(\alpha\sqrt{(x-a)^2+y^2})\right]d\alpha. \tag{5}$$

The current density at the midplane, $x=0$, will therefore, by Eq. (1), have the form:

$$i = 2a\sigma\int_0^\infty \frac{\alpha J_1(\alpha\sqrt{a^2+y^2})F(\alpha, z)}{\sqrt{a^2+y^2}}\,d\alpha. \tag{6}$$

Introducing this value into Eq. (2), evaluating the integral with respect to y, and then dividing by the total input current I, we thus obtain for the fraction of the current passing between the plane z_1 and z_2:

$$f = \frac{4\sigma}{I}\int_0^\infty \sin\alpha a d\alpha \int_{z_1}^{z_2} F(\alpha, z)dz. \tag{7}$$

While one cannot write out in advance the general expression for the integral with respect to z, because $F(\alpha, z)$ has no universal form, it will always be a simple matter to evaluate the integral in any special case, as the terms in F depending on z will always be of an exponential character. Moreover these exponential terms will always insure the convergence of the resulting integrals with respect to α, except for such specific cases where one of the strata has zero conductivity. The latter, however, can be treated separately without difficulty, as will be seen below.

[3] M. Muskat, "Potential Distribution about an Electrode on the Surface of the Earth," Physics *4*, 129 (1933).

Before applying Eq. (7) to some special cases, it is of interest to note that if instead of Eq. (4) the image representation for the potential distribution be used, the resultant value for f can be formally written out explicitly. Thus supposing that the image system appropriate to the particular kind of stratification under consideration consists of images of strength q_n at distances b_n along the z axis, the resultant potential distribution will be:

$$\Phi = \frac{1}{\sqrt{(x+a)^2+y^2+z^2}} - \frac{1}{\sqrt{(x-a)^2+y^2+z^2}} + \sum q_n \left[\frac{1}{\sqrt{(x+a)^2+y^2+(z-b_n)^2}} - \frac{1}{\sqrt{(x-a)^2+y^2+(z-b_n)^2}} \right]. \tag{8}$$

This will give a current density at $x=0$ of:

$$i = 2\sigma a \left[\frac{1}{(a^2+y^2+z^2)^{3/2}} + \sum \frac{q_n}{\{a^2+y^2+(z-b_n)^2\}^{3/2}} \right].$$

Noting that the total current input corresponding to Φ is $2\pi\sigma_0$, where σ_0 is the surface conductivity, we obtain directly for the fraction passing between z_1 and z_2:

$$f = \frac{2\sigma}{\pi\sigma_0} \left[\tan^{-1}\frac{z_2}{a} - \tan^{-1}\frac{z_1}{a} + \sum q_n \left\{ \tan^{-1}\frac{z_2-b_n}{a} - \tan^{-1}\frac{z_1-b_n}{a} \right\} \right]. \tag{9}$$

A simple interpretation of Eq. (9) follows from the observation that the differences of the pairs of arctangent terms are really nothing more than the angles in the x, z plane, with vertices at the images, subtended by the section z_1z_2. Denoting these angles by θ_n, we may rewrite Eq. (9) as:

$$f = \frac{2\sigma}{\pi\sigma_0} \sum q_n\theta_n, \tag{10}$$

where the direct effect of the original sources is here also included in the sum. Thus each source pair—real or virtual—contributes to the current fraction through any section an amount equal to $2\sigma/\pi\sigma_0$ times

the product of its strength and the plane angle subtended by the section with either source as the vertex.

While this formula is interesting from a physical pointof view in that it separates out the contributions due to the individual images, the location of their positions, b_n, and particularly their strengths, q_n, is in general a very tedious process. We shall therefore use in the following Eq. (7) for f and Eq. (4) for Φ.

THE TWO-LAYER EARTH

Of particular interest for the problem of the two-layer earth will be the total fractions of current passing through either the top layer or the bottom layer, as indicated in Fig. 2. Moreover as their sum will

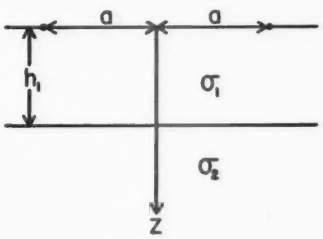

FIG. 2

necessarily equal unity, we need concern ourselves directly only with either one of the two. For definiteness we shall therefore restrict ourselves to the top layer. For this case it has been shown[3] that $F(a,z)$ has the form:

$$\left.\begin{aligned} F(\alpha, z) &= \frac{2h_1\left[e^{-\alpha z} + ke^{-2\alpha h_1+\alpha z}\right]}{1 - ke^{-2\alpha h_1}}, \\ \text{where:} \qquad k &= \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2}, \end{aligned}\right\} \tag{11}$$

and the upper layer thickness is h_1. We then have on inserting Eq. (11) into Eq. (7):

$$f_1 = \frac{8\sigma_1 h_1}{I}\int_0^\infty \frac{\sin \alpha a d\alpha}{\alpha}\left[1 - \frac{(1-k)e^{-\alpha h_1}}{1 - ke^{-2\alpha h_1}}\right]. \tag{12}$$

Noting now that the total input current corresponding to the $F(\alpha,z)$ of Eq. (11) is given by:

$$I = 4\pi\sigma_1 h_1,$$

Eq. (12) reduces to:

$$f_1 = 1 - \frac{2}{\pi}(1 - k)\int_0^\infty \frac{e^{-\alpha h_1}\sin \alpha a d\alpha}{\alpha(1 - ke^{-2\alpha h_1})}. \tag{14}$$

which may also be expressed as:

$$\begin{aligned} f_1 &= 1 - \frac{2}{\pi}(1 - k)\sum_0^\infty k^n \tan^{-1}\frac{a/h_1}{2n+1} \\ &= 1 - \frac{4\sigma_2/\sigma_1}{\pi(1+\sigma_2/\sigma_1)}\sum_0^\infty \left[\frac{1-\sigma_2/\sigma_1}{1+\sigma_2/\sigma_1}\right]^n \tan^{-1}\frac{a/h_1}{2n+1}. \end{aligned} \tag{15}$$

The series term represents, of course, f_2, that is, the fraction of the input current passing through the bottom layer.

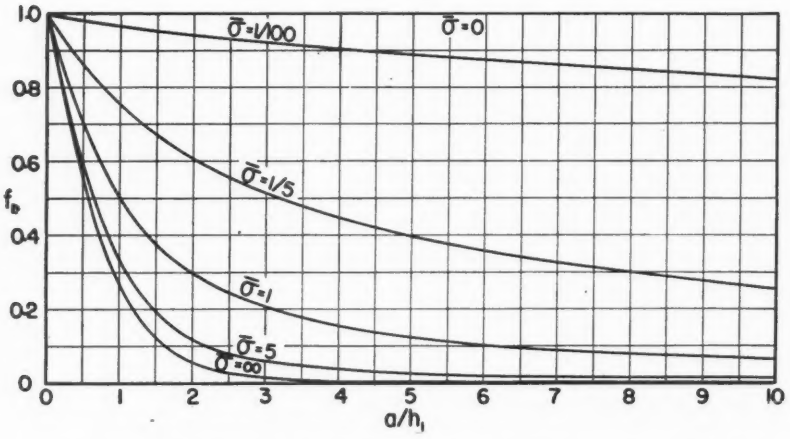

FIG. 3. Current fractions, f_1, in the surface stratum of a two-layer earth. a/h_1 = (half of current electrode spacing)/(surface layer thickness); $\bar{\sigma} = \sigma_2/\sigma_1$ = (conductivity of second layer)/(conductivity of surface layer).

As is to be expected, f_1 depends only upon the dimensionless ratios σ_2/σ_1 and a/h_1. In Fig. 3 are plotted several curves for f_1 calculated by means of Eq. (15) and using these ratios as the parameters. For small conductivity contrasts, that is, when σ_2/σ_1 does not differ greatly from 1, the series of Eq. (15) converges rapidly without further manipulation. For greater contrasts, and especially for k approaching the values ± 1, it is convenient to transform Eq. (15) into:

$$f_1 = 1 - \frac{a}{\pi h_1}\left(\frac{1-k}{\sqrt{k}}\right)\log\frac{1+\sqrt{k}}{1-\sqrt{k}} - \frac{2}{\pi}(1-k)\sum_0^\infty k^n\left(\tan^{-1}\frac{a/h_1}{2n+1} - \frac{a/h_1}{2n+1}\right), \tag{16}*$$

the series in which converges as $k_n/(2n+1)^3$. When $\sigma_2 > \sigma_1$, k is negative and the series is alternating, thus improving still further the convergence.

For the special case where $\sigma_2 = \sigma_1$ and $k = 0$, Eq. (15) reduces to:

$$f_1 = 1 - \frac{2}{\pi}\tan^{-1}\frac{a}{h_1} = \frac{2}{\pi}\tan^{-1}\frac{h_1}{a}, \tag{17}$$

which, as should be expected, gives the fractional penetration up to the depth h_1 in a homogeneous earth. This is the formula that was presumably used by Weaver[2] in his plot of the penetration curve. It is also of interest to note that the arctangent series of Eq. (15) is really nothing more than the special form of the series of Eq. (9) given directly by the image representation for the potential, but with the image locations and strengths automatically provided by the Bessel function representations of Eqs. (4) and (7).

When the bottom layer is an insulator, that is, $\sigma_2 = 0$, $k = 1$, and both the integral in Eq. (14) and the series in Eq. (15) for f_1 diverge. This corresponds to the well known fact that the potential distribution under such conditions as given orginally by Eq. (4) and by the equivalent image series of Eq. (8) also diverge. For the present purposes, however, this is of no consequence, as it is clear physically that when the bottom layer is an insulator all of the current must pass through the top stratum, so that $f_1 = 1$ for all values of a or h_1. Moreover, it may actually be shown analytically that the factor $1-k$ before the integral of Eq. (14) cancels the divergence and gives directly $f_1 = 1$.

In the other limiting case when the lower stratum has an infinite conductivity, $k = -1$. While the series representation of Eq. (15) does not then diverge, it is not convenient to handle numerically. A closed expression, however, may then be obtained by returning to Eq. (14) and setting $k = -1$ directly, which gives:

$$f_1 = 1 - \frac{2}{\pi}\int_0^\infty \frac{\sin\alpha a}{\alpha\cosh\alpha h_1}\,d\alpha = \frac{4}{\pi}\tan^{-1}e^{-\pi a/2h_1}. \tag{18}$$

* For $k<0$ the second term may be expressed as: $-\frac{2a}{\pi h_1}\frac{(1-k)}{\sqrt{|k|}}\tan^{-1}\sqrt{|k|}$.

It is thus seen that while the current flowing through the first layer dies off exponentially as the electrode spacing is increased, it will still take the major part of the current when a/h_1 becomes small, even though the bottom layer has a vanishing resistivity.

For large values of a/h_1 the series of Eqs. (15) and (16) converge very slowly. It is then convenient to use for f_1 an asymptotic expansion in terms of the spacing to thickness ratio, a/h_1. This may be obtained from Eqs. (7) and (11), and, to terms in $(h_1/a)^5$, one finds:

$$f_1 = \frac{2}{\pi}\frac{\sigma_1}{\sigma_2}\frac{h_1}{a}\left[1 + \frac{h_1^2}{3a^2}\left(5 - \frac{6\sigma_1^2}{\sigma_2^2}\right) + \frac{h_1^4}{5a^4}\left(120\,\frac{\sigma_1^4}{\sigma_2^4} - 180\,\frac{\sigma_1^2}{\sigma_2^2} + 61\right) + \cdots\right]. \tag{19}$$

This formula is quite satisfactory for large values of a/h_1, and for σ_1/σ_2 not appreciably greater than 1. It may be readily verified that when $\sigma_1/\sigma_2 = 1$ Eq. (19) gives the first three terms of the expansion of the closed rigorous value of f_1 for a homogeneous earth.

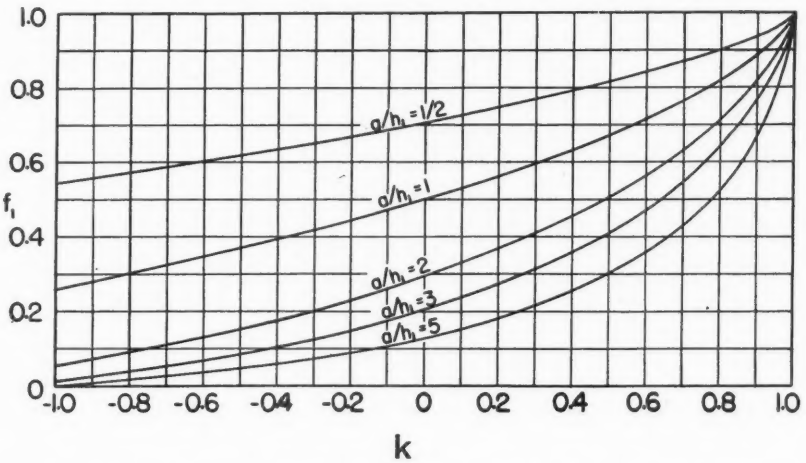

FIG. 4. Current fractions, f_1, in the surface stratum of a two-layer earth. a/h_1 = (half of current electrode spacing)/(surface layer thickness); $k = (1-\bar{\sigma})/(1+\bar{\sigma})$ where $\bar{\sigma}$ = (conductivity of second layer)/(conductivity of surface layer).

The curves of Fig. 3 show that, as is to be expected, the current fraction passing through the upper layer of the two-layer earth decreases uniformly as the electrode spacing is increased or the upper layer thickness decreases. Moreover the rate of this decrease becomes

greater as σ_2/σ_1 increases, that is, as the conductivity of the lower stratum increases relatively to the upper layer. However, even when the lower stratum is of infinite conductivity the current flowing through the upper layer will still be an appreciable fraction of the total current until the current electrode spacing exceeds six times the upper layer thickness.

The variation of f_1 with the conductivity parameter k for different electrode spacings is shown in Fig. 4. It will be noted that the variation of k between -1 and $+1$ covers the whole range of variation of the actual conductivity ratios σ_2/σ_1. The continuous decrease of f_1 with decreasing k again simply implies a continuously decreasing current fraction in the upper layer as the conductivity of the lower stratum increases. It is of interest, however, to note the markedly different slopes in the curves at $k=-1$ and $k=+1$. This indicates, as may also be verified by reference to Fig. 3, that the current fraction passing through the upper stratum is much more sensitive to the absolute value of the lower stratum conductivity when the resistance of the latter is relatively high than when that stratum is a highly conducting bed. From the point of view of actual prospecting by resistivity measurements, this feature of the current penetration curves clearly explains the well known fact that the sensitivity of detecting a poorly conducting bed will be greater than that for the detection of a highly conducting bed.

THE THREE LAYER EARTH

For the case of the three layer earth, such as represented diagrammatically in Fig. 5, one must consider the potential distribution func-

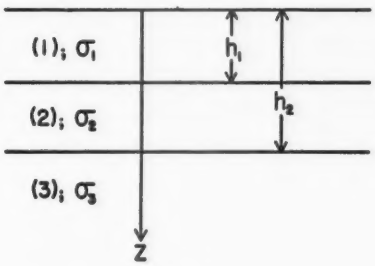

FIG. 5

tions in at least two of the layers in order to obtain a complete picture of the current penetrations. For completeness we shall list all three.

These may be obtained by straightforward potential theory methods and expressed in the form of Eq. (4). Denoting by subscripts the three individual layers, it is found that the functions $F(\alpha, z)$, entering in Eq. (4) to give the potential distribution in the three layers, are:

$$\left.\begin{aligned} F_1 &= \frac{e^{-\alpha z} + k_2 e^{-\alpha(2h_1 - z)} + k_3 e^{-\alpha(2h_2 - z)} + k_2 k_3 e^{-\alpha(2h_2 - 2h_1 + z)}}{\Delta}, \\ F_2 &= \frac{(1 + k_2)(e^{-\alpha z} + k_3 e^{-\alpha(2h_2 - z)})}{\Delta}, \\ F_3 &= \frac{(1 + k_2)(1 + k_3) e^{-\alpha z}}{\Delta}, \end{aligned}\right\} \tag{20}$$

where:

and:

$$\left.\begin{aligned} \Delta &= 1 - k_2 e^{-2\alpha h_1} - k_3 e^{-2\alpha h_2} + k_2 k_3 e^{-2\alpha(h_2 - h_1)}, \\ k_2 &= \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2}; \qquad k_3 = \frac{\sigma_2 - \sigma_3}{\sigma_2 + \sigma_3}, \end{aligned}\right\} \tag{21}$$

and the notations with respect to h_1, h_2, σ_1, σ_2, and σ_3 are those indicated in Fig. 5.

Carrying out the integrations of the F functions with respect to z, as indicated in Eq. (7), subtracting off from F_1 the contribution corresponding to the surface electrode, and introducing the notation:

$$\bar{a} = a/h_1; \qquad h = h_2/h_1, \tag{22}$$

it is then found that the current fractions f take the form:

$$\left.\begin{aligned} f_1 &= \frac{2}{\pi} \tan^{-1} \frac{1}{\bar{a}} + \frac{4}{\pi} \int_0^\infty \frac{[k_2 e^{-2\alpha} + k_3 e^{-2\alpha h}] \sinh \alpha \sin \alpha \bar{a} d\alpha}{\alpha \Delta} \\ f_2 &= \frac{2}{\pi} (1 - k_2) \int_0^\infty e^{-\alpha h} \frac{[k_3 - 1 + e^{+\alpha(h-1)} - k_3 e^{-\alpha(h-1)}] \sin \alpha \bar{a} d\alpha}{\alpha \Delta} \\ f_3 &= \frac{2(1 - k_2)(1 - k_3)}{\pi} \int_0^\infty \frac{e^{-\alpha h} \sin \alpha \bar{a} d\alpha}{\alpha \Delta} \end{aligned}\right\} \tag{23}$$

where now:

$$\Delta = 1 - k_2 e^{-2\alpha} - k_3 e^{-2\alpha h} + k_2 k_3 e^{-2\alpha(h-1)},$$

where the total current flow I has been taken as $2\pi\sigma_1$, in accordance with the value of F_1 of Eq. (20).

We shall now apply these formulas to a number of specific types of conductivity stratification.

INTERMEDIATE STRATA IN HOMOGENEOUS EARTHS

For the first type of stratification which we shall consider numerically we shall take two cases where the earth as a whole will be assumed homogeneous except for an intermediate layer. That is, we shall take $\sigma_1=\sigma_3$, leaving the ratio σ_2/σ_1 as the only conductivity contrast in the system. Even with this simplification, however, it is very difficult to obtain convenient numerical representations of the fractions f for general arbitrary values of the depth ratio h, and one must derive separate evaluations of the integrals of Eq. (23) for the individual values of h. In the present section we shall treat the two cases where $h=2$ and $h=3$, which correspond to systems in which the middle layer thickness is equal to or twice that of the surface stratum, respectively. The treatment for $h=2$ is as follows:

$h=2$

Because of our assumption that the surface and the deepest layers will have the same conductivity we may set at once:

$$\left.\begin{aligned} k_2 &= -k_3 \equiv k, \\ \Delta &= 1 - k(1+k)e^{-2\alpha} + ke^{-4\alpha}. \end{aligned}\right\} \tag{24}$$

f_1 then takes the form:

$$f_1 = \frac{2}{\pi}\tan^{-1}\frac{1}{\bar{a}} + \frac{8k}{\pi}\int_0^\infty \frac{e^{-3\alpha}\sin\alpha\bar{a}\sinh^2\alpha d\alpha}{\alpha\Delta}. \tag{25}$$

Now with the above form for Δ, it may be readily shown that:

$$\left.\begin{aligned} \frac{e^{-2\alpha}}{\Delta} &= \frac{1}{s_2-s_1}\left[\frac{1}{1-s_2e^{-2\alpha}} - \frac{1}{1-s_1e^{-2\alpha}}\right] \\ &= \frac{1}{s_2-s_1}\sum_1^\infty (s_2{}^n - s_1{}^n)e^{-2n\alpha}, \end{aligned}\right\} \tag{26}*$$

where:

$$\left.\begin{aligned} 2s_1 &= k(1+k) - \sqrt{k^2(1+k)^2 - 4k} \\ 2s_2 &= k(1+k) + \sqrt{k^2(1+k)^2 - 4k}. \end{aligned}\right\}$$

* The same type of expansion, for $h=2$, can be used for general values of k_2 and k_3, with obvious changes in the expressions for s_1 and s_2.

Inserting the expansion of Eq. (26) into the integral for f_1 one immediately obtains:

$$f_1 = \frac{2}{\pi}\left[\tan^{-1}\frac{1}{\bar{a}} + \frac{k}{s_2 - s_1}\sum_1^\infty (s_2{}^n - s_1{}^n)\left\{\tan^{-1}\frac{\bar{a}}{2n-1}\right.\right. \tag{27}$$

$$\left.\left. - 2\tan^{-1}\frac{\bar{a}}{2n+1} + \tan^{-1}\frac{\bar{a}}{2n+3}\right\}\right].$$

Subtracting from the arctangents their arguments, as was done in the case of Eq. (16) so as to improve the convergence of the series, and, summing separately the remaining series, one finally obtains:

$$f_1 = \frac{2}{\pi}\left[\tan^{-1}\frac{1}{\bar{a}} + \frac{k\bar{a}}{s_2 - s_1}\left\{ \left(1 - \frac{1}{s_2}\right)^2 \frac{\sqrt{s_2}}{2}\log\frac{1+\sqrt{s_2}}{1-\sqrt{s_2}} - \frac{1}{s_2}\right.\right.$$

$$\left. + \left(1 - \frac{1}{s_1}\right)^2 \sqrt{-s_1}\tan^{-1}\sqrt{-s_1} + \frac{1}{s_1}\right\}$$

$$- \frac{k}{s_2 - s_1}\sum_1^\infty (s_2{}^n - s_1{}^n)\left\{\left(\frac{\bar{a}}{2n-1} - \tan^{-1}\frac{\bar{a}}{2n-1}\right)\right. \tag{28}$$

$$- 2\left(\frac{\bar{a}}{2n+1} - \tan^{-1}\frac{\bar{a}}{2n+1}\right)$$

$$\left.\left. + \left(\frac{\bar{a}}{2n+3} - \tan^{-1}\frac{\bar{a}}{2n+3}\right)\right\}\right].$$

where now the series converges as s^n/n^5.

The above series may be used directly for $k<0$, when both s_1 and s_2 are real. For $k>0$, s_1 and s_2 are complex conjugates, and may be expressed as:

$$s_1 = Re^{-i\theta}; \qquad s_2 = Re^{i\theta}. \tag{29}$$

On introducing these representations into the original expansions, or directly into Eq. (28), one then obtains:

$$
\left.\begin{aligned}
f_1 = \frac{2}{\pi}\Bigg[& \tan^{-1}\frac{1}{\bar{a}} + \bar{a} \\
& - \frac{k^{1/4}\bar{a}(1-k)}{2R\sin\theta}\left\{(1+\sqrt{k})\cos\frac{\theta}{2}\tan^{-1}\frac{2\sqrt{R}\sin\theta/2}{1-R}\right. \\
& \left. + \frac{(1-\sqrt{k})\sin\theta/2}{2}\log\frac{1-2R\cos\theta+R^2}{\left(1-2\sqrt{R}\cos\frac{\theta}{2}+R\right)^2}\right\} \\
& - \frac{k}{R\sin\theta}\sum_1^\infty R^n \sin n\theta\left\{\left(\frac{\bar{a}}{2n-1}-\tan^{-1}\frac{\bar{a}}{2n-1}\right)\right. \\
& - 2\left(\frac{\bar{a}}{2n+1}-\tan^{-1}\frac{\bar{a}}{2n+1}\right) \\
& \left. + \left(\frac{\bar{a}}{2n+3}-\tan^{-1}\frac{\bar{a}}{2n+3}\right)\right\}\Bigg].
\end{aligned}\right\} \tag{30}
$$

Because of the fact that the sum of the three f's must necessarily equal unity, it is sufficient to evaluate only one of the remaining two f's in order to obtain the complete current distribution. For this purpose it is more convenient to evaluate f_3. In order to obtain more satisfactory convergence in the final series expansions, it is useful to first subtract off an arctangent term as indicated by the expression:

$$
f_3 = \frac{2}{\pi}\tan^{-1}\frac{\bar{a}}{2} - \frac{2}{\pi}\int_0^\infty \frac{\sin\alpha\bar{a}}{\alpha}\,\frac{\left[k^2-k(1+k)e^{-2\alpha}+ke^{-4\alpha}\right]e^{-2\alpha}d\alpha}{\Delta}. \tag{31}
$$

Treating the resulting integral in a manner similar to that outlined in the disucussion of f_1, it is then readily found that f_3 will be given by:

$$
\left.\begin{aligned}
f_3 = \frac{2}{\pi}\tan^{-1}\frac{\bar{a}}{2} & - \frac{k\bar{a}}{\pi s_1^2 s_2^2(s_2-s_1)}\left[\left\{ks_1^2-(1+k)s_1+1\right\}s_2^2\log(1-s_1)\right. \\
& \left. - \left\{ks_2^2-(1+k)s_2+1\right\}s_1^2\log(1-s_2)+s_1s_2(s_2-s_1)\right] \\
& + \frac{2k}{\pi(s_2-s_1)}\sum_1^\infty (s_2^n-s_1^n)\left[k\left(\frac{\bar{a}}{2n}-\tan^{-1}\frac{\bar{a}}{2n}\right)\right. \\
& - (1+k)\left(\frac{\bar{a}}{2n+2}-\tan^{-1}\frac{\bar{a}}{2n+2}\right) \\
& \left. + \left(\frac{\bar{a}}{2n+4}-\tan^{-1}\frac{\bar{a}}{2n+4}\right)\right].
\end{aligned}\right\} \tag{32}
$$

This may be used directly for $k<0$. For $k>0$ it may be transformed into:

$$\left.\begin{aligned}\frac{\pi}{2}f_3=\tan^{-1}\frac{\bar{a}}{2}-\frac{\bar{a}}{2}+\frac{1}{R\sin\theta}\Bigg[\frac{\bar{a}}{2}(1-k^2)\tan^{-1}\frac{R\sin\theta}{1-R\cos\theta}\\ +k\sum_1^\infty R^n\sin n\theta\Bigg\{k\left(\frac{\bar{a}}{2n}-\tan^{-1}\frac{\bar{a}}{2n}\right)\\ -(1+k)\left(\frac{\bar{a}}{2n+2}-\tan^{-1}\frac{\bar{a}}{2n+2}\right)\\ +\left(\frac{\bar{a}}{2n+4}-\tan^{-1}\frac{\bar{a}}{2n+4}\right)\Bigg\}\Bigg]\end{aligned}\right\}\quad(33)$$

The series in these expressions converges as $(1-k)/n^4$.

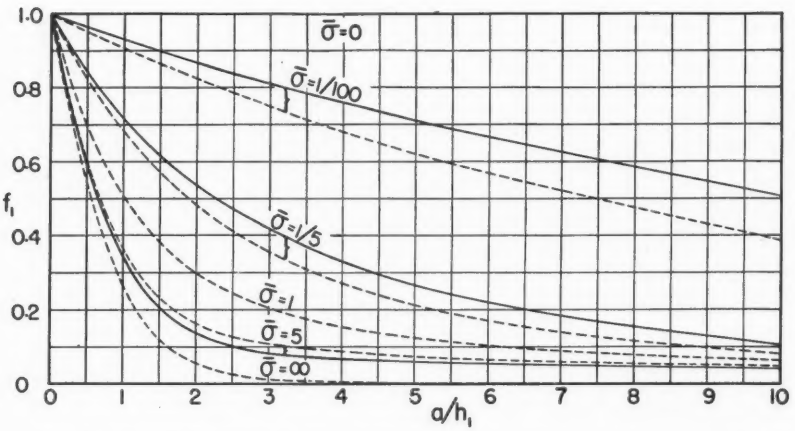

FIG. 6. Current fractions, f_1, in the surface stratum of a three-layer earth, having the same conductivity in the first and third layers. ..... : middle layer thickness = surface layer thickness, ($h=2$); —— : middle layer thickness = 2 · surface layer thickness, ($h=3$). Rest of notation as in Fig. 3.

The numerical results calculated by means of the above formulas are shown graphically as the dotted curves in Figs. 6–10. In Fig. 6 are plotted the curves giving the fraction of the total current passing through the uppermost layer as a function of the electrode spacing, or top layer thickness, for several values of the conductivity contrast. These curves correspond to and are similar to those shown in Fig. 3 for the two-layer earth. There are, however, two major differences.

These are: (1), that for conductivity ratios $\sigma_2/\sigma_1 < 1$ the current fractions f_1 in the uppermost layer are uniformly lower in the three-layer earth than in the two-layer earth; (2), when $\sigma_2/\sigma_1 > 1$ the converse is true. That is, in that case the current fractions in the upper stratum of the three-layer earth exceed those in the two-layer earth. The reason for these differences is quite simple when one recalls that the three-layer earth referred to in Figs. 6–10 is one in which the deepest layer has the same conductivity as the surface stratum. In the present case when the second stratum has a relatively low conductivity the high

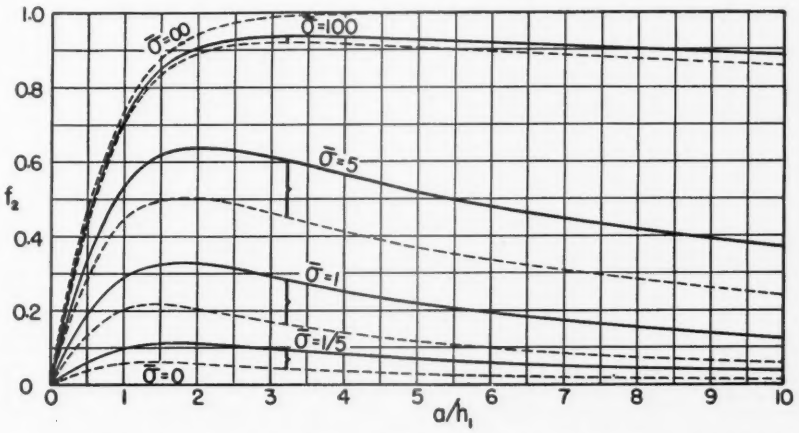

FIG. 7. Current fractions, f_2, in the middle stratum of a three-layer earth, having the same conductivity in the first and third layers. Notation as in Fig. 6.

conductivity of the third layer would naturally attract more of the surface current to the deeper levels. This would decrease f_1 more than in the case of the two-layer earth where the low conductivity of the second bed extends to infinite depths. On the other hand, when the second stratum conductivity is high the relatively low conductivity of the third stratum will in effect discourage excessive penetration of the surface current. Thus it will force a larger fraction of the input current to pass through the uppermost layer than in the two-layer earth where the highly conducting second bed is infinitely thick. Except for these differences the general decline in the current fractions f_1 with increasing electrode spacings or decreasing top layer thickness has the same interpretation for the case of the three-layer earth as for the two-layer earth.

The fractions of the current passing through the second layer which

were obtained by subtracting from unity the fractions f_1 and f_3 calculated by means of the equations given above, are plotted as the dotted curves in Fig. 7. The most striking feature about these curves is that, in contrast to those for f_1, they show maxima. That is, whereas the fractions f_1 monotonically decrease with increasing values of a/h_1, those for f_2 first increase, then reach maxima, and finally fall off again as a/h_1 is still further increased. This behavior, is, however, quite reasonable. Thus the initial rise simply expresses the increasing overall current penetration as the electrode spacing is widened relatively to

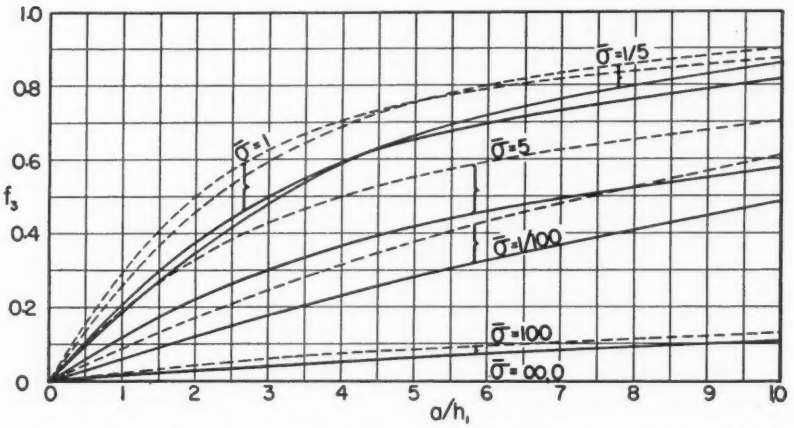

FIG. 8. Current fractions, f_3, in the deepest stratum of a three-layer earth, having the same conductivity in the first and third layers. Notation as in Fig. 6.

the top layer thickness. The ultimate declines in the curves and intermediate maxima merely expresses the fact that as the ratio a/h_1 continues to grow the current becomes largely concentrated in the deepest or third layer. The same considerations also explain the fact that the values of a/h_1, at which the maxima appear, increase with increasing σ_2/σ_1. For as the middle layer conductivity is increased relative to that of the remaining strata, it is natural that it would tend to hold longer the concentration of current through it as a/h_1 is increased, than when its conductivity is relatively lower. Of course, in the limiting case when the middle layer has an infinite conductivity it blankets the bottom layer completely and the maximum in f_2 is reached asymptotically without developing any decline or loss of current to the deepest layer. Likewise when the second layer has zero conductivity the current through it will, of course, be always nil regardless of the value of a/h_1.

Fig. 8, giving the curves—dotted for $h=2$—for f_3, really represents nothing more than the differences from unity of the sums of the ordinates of the curves of Figs. 6 and 7. Here the monotonic rise of the curves with increasing a/h_1 is again simply an expression of the predominant characteristic of all current distribution systems through continuous and stratified earths, namely, that of increasing current penetration with increasing electrode spreads. It is also to be noted here that as the conductivity of the middle layer decreases from infinity, relative to that of the others, the current fraction f_3 increases

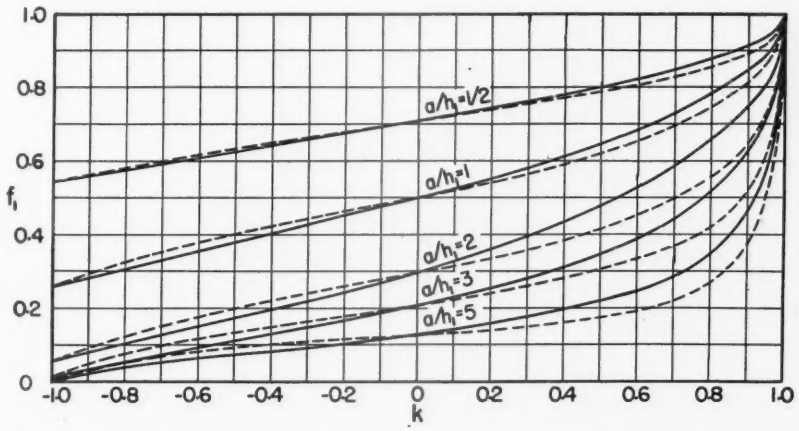

FIG. 9. Current fractions, f_1, in the surface stratum of a three-layer earth, having the same conductivity in the first and third layers. Notation as in Figs. 4 and 6.

simply because of a lessening of the blanketing effect of the conducting middle layer. With further decreases in ratio of σ_2/σ_1 the fraction f_3 ultimately reaches a maximum and then falls off once more to zero as σ_2/σ_1 becomes zero, and the middle layer now becomes an insulating blanket with respect to the deepest layer. The variation of the conductivity ratio at which the maximum is reached with the value of a/h_1 clearly is the cause of the crossing of the two uppermost dotted curves of Fig. 8.

In Fig. 9 are plotted the current fractions f_1 as functions of the conductivity parameter k, for the various values of the ratio a/h_1. The dotted curves are here, too, those for $h=2$. Again, as a whole, these curves are very similar to those of Fig. 4 for the two-layer earth. The most striking difference lies in the markedly steeper rates of decline in the curves as k becomes less than unity. While theoretically the

slopes of the f_1 curves become logarithmically infinite at the limit $k=1$ both for the two-layer and three-layer earths, the greater persistence of the higher slopes for the three-layer earth is clearly due to the highly conducting third layer which is enabled to make its presence felt as soon as the middle layer is given any conductivity at all. At the other limit for the range of k, namely, $k=-1$, the curves in Fig. 9 show a convexity in contrast to those of Fig. 4 which remain concave throughout the complete range of k. This difference arises from the fact that although the f_1 values for $k<0$ are in general higher for the three-layer

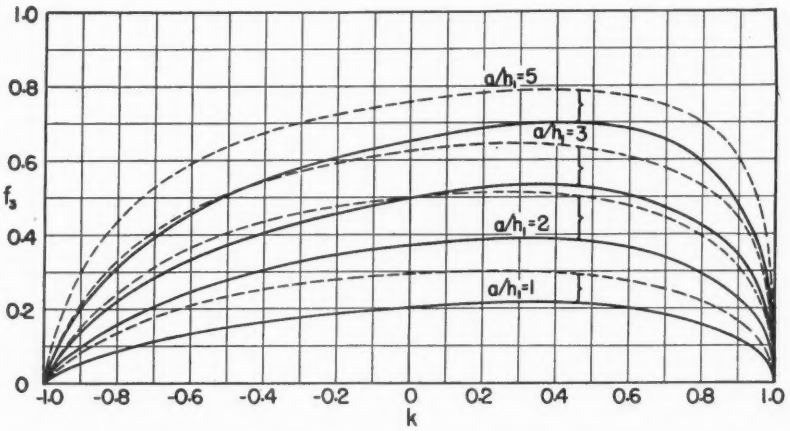

FIG. 10. Current fractions, f_3, in the deepest stratum of a three-layer earth, having the same conductivity in the first and third layers. Notation as in Figs. 4 and 6.

earth than for the two-layer earth, as previously indicated, they must ultimately become equal at the limit $k=-1$, where the middle layer forms a conducting blanket and hence completely masks the presence of the third layer.

The corresponding curves for f_2 vs. k are so closely intertwined as to contribute nothing to the physical interpretation of the problem. Those for f_3 are shown in Fig. 10. Here the steep rises in the curves as k becomes less than 1 simply represent the complementary effect of the sharp falls in the f_1 curves of Fig. 9, the interpretation of which has already been given. And again because of the blanketing effect of the middle layer as its conductivity becomes infinite, the current fractions f_3 must necessarily fall to zero as $k\rightarrow-1$. The intermediate maxima are thus but obvious consequences of the behavior at the two limits of the range of k. Finally, the variation in the relative positions of the

curves for the different values of a/h_1 in Fig. 10, and the converse variation in Fig. 9, are clearly again nothing more than expressions of the general increase of current penetration with increasing electrode spacing.

$h=3$

Because of the great complexity of the detailed analysis of this case we shall only give an outline of the method of evaluation of the current fractions. The individual steps and explicit intermediate formulas can be easily filled in by the reader. The actual integral expressions to be evaluated are those given by:

$$\left.\begin{aligned} f_1 &= \frac{2}{\pi}\tan^{-1}\frac{1}{\bar{a}} + \frac{2k}{\pi}\int_0^\infty \frac{e^{-\alpha}\sin\alpha\bar{a}}{\alpha\Delta}\left[1 - e^{-2\alpha} - e^{-4\alpha} + e^{-6\alpha}\right]d\alpha \\ f_2 &= \frac{2}{\pi}(1-k)\int_0^\infty \frac{e^{-\alpha}\sin\alpha\bar{a}}{\alpha\Delta}\left[1 - (1+k)e^{-2\alpha} + ke^{-4\alpha}\right]d\alpha \\ f_3 &= \frac{2}{\pi}(1-k^2)\int_0^\infty \frac{e^{-3\alpha}\sin\alpha\bar{a}}{\alpha\Delta}\,d\alpha \end{aligned}\right\} \quad (34)$$

where:

$$\Delta = 1 - ke^{-2\alpha} - k^2e^{-4\alpha} + ke^{-6\alpha}.$$

The first step is that of factoring the denominator Δ. This process gives:

$$\frac{1}{\Delta} = \frac{1}{k(s_3-s_1)(s_3-s_2)(s_2-s_1)}\left[\frac{s_3-s_2}{e^{-2\alpha}-s_1} + \frac{s_1-s_3}{e^{-2\alpha}-s_2} + \frac{s_2-s_1}{e^{-2\alpha}-s_3}\right], \quad (35)$$

where the s's are the roots of cubic equations determing the zeros of Δ. It may be shown that one of these, s_1, for example, is real and the other two complex conjugates.

The complete problem of evaluating the current fractions clearly reduces to that of determining the general integrals defined by:

$$I_m = \int_0^\infty \frac{e^{-m\alpha}\sin\alpha\bar{a}}{\alpha\Delta}\,d\alpha, \quad (36)$$

which can in turn be split, in view of Eq. (35), into the sum of the separate integrals:

$$I_m = F_m(\beta_1) + F_m(\beta_2) + F_m(\beta_3),$$

where:

$$F_m(\beta) = A(\beta)\int_0^\infty \frac{e^{-m\alpha}\sin\alpha\bar{a}}{\alpha(1-\beta e^{-2\alpha})}\,d\alpha, \quad (37)$$

the relations between the β's and s's and the values of the coefficients $A(\beta)$ being easily derived from the preceding formulas. It is to be noted, however, that β_1 and $A(\beta_1)$ will be real whereas β_2 and β_3 will be complex conjugates, as will also $A(\beta_2)$ and $A(\beta_3)$.

The general expression for $F_m(\beta)$ may readily be verified to be:

$$\begin{aligned} F_m(\beta) &= A(\beta) \sum_0^\infty \beta^n \tan^{-1} \frac{\bar{a}}{2n+m} \\ &= \frac{1}{\beta}\left[F_{m-2}(\beta) - A(\beta) \tan^{-1} \frac{\bar{a}}{m-2} \right]. \end{aligned} \tag{38}$$

On subtracting out the arguments of the arctangent terms and summing these separately, as was done in passing from Eq. (15) to Eq. (16), the $F_m(\beta_1)$ can be thus evaluated directly with an improved series convergence. For the other two F_m functions, it is convenient to express β_2 and β_3 in complex forms as:

$$\beta_2 = Re^{i\theta}; \qquad \beta_3 = Re^{-i\theta}, \tag{39}$$

and treat $F_m(\beta_2)$ and $F_m(\beta_3)$ together. One thus finds at once:

$$\left.\begin{aligned} F_m(\beta_2)+F_m(\beta_3) = \{A(\beta_2)+A(\beta_3)\} \sum_0^\infty R^n \cos n\theta \tan^{-1} \frac{\bar{a}}{2n+m} \\ +i\{A(\beta_2)-A(\beta_3)\} \sum_1^\infty R^n \sin n\theta \tan^{-1} \frac{\bar{a}}{2n+m}. \end{aligned}\right\} \tag{40}$$

On again subtracting from the arc tangent terms their arguments and summing these terms separately one will obtain more rapidly converging remaining series quite similar to those of Eqs. (30) and (33). When these are added to the evaluations of $F_m(\beta_1)$ the resultant values of I_m will be obtained and by linear composition in appropriate forms will give directly the final numerical results for the fractions f_1, f_2, and f_3.

The numerical results obtained in the manner just outlined are plotted as the solid curves of Figs. 6–10. As these curves are quite similar to those for $h=2$, the dotted curves, we need not discuss again the variation with the abscissae variables a/h_1 or k. Rather we shall note here only the differences between the dotted and solid curves which may reflect the effect of the variation in intermediate stratum thickness. Thus in Fig. 6 it will be noted that for $\sigma_2/\sigma_1 < 1$ the solid curves

lie higher than the dotted ones, whereas the converse is true for $\sigma_2/\sigma_1 > 1$. This is quite reasonable, since the middle layer of low conductivity would naturally prevent the penetration of the surface current to an extent increasing with the thickness of the stratum, whereas when the intermediate stratum is highly conducting it will draw down the surface currents in increasing amounts as its thickness increases. The behavior indicated in Fig. 6 follows at once from this observation. Of course, for $\sigma_2/\sigma_1 = 1$, the solid curves would become identical with that shown as dotted, since for this case the earth is homogeneous and the middle stratum thickness does not affect the current fractions in the upper layer.

In the case of the current fractions f_2, the curves for the thick middle layer uniformly overlie those for the thinner middle layer. This is obviously nothing more than the effect of the absolute thickness of the middle layer, although it is to be noted that the variations of the current fractions f_2 are not as great as those in the absolute thickness of the stratum.

The variation of f_3 with a/h_1 is shown in Fig. 8. Here it will be seen that in all cases the current passing through the deepest layer will be smaller when the middle stratum is thick than when it is relatively thin. This is a consequence of two factors. First, as the middle layer thickness is increased the absolute depth of the third stratum becomes greater and the total current through it inherently tends to decrease. Secondly, the shielding effect of the middle stratum either as an insulating or conducting blanket will naturally increase as the middle layer thickness is made larger, thus again leading to smaller current fractions in the deepest layer.

The variation of f_1 with k for $h = 3$ is shown by the solid curves of Fig. 9. Here the solid and dotted curves of each pair all cross at $k = 0$. For $\sigma_2/\sigma_1 < 1$ the fractions f_1 for $h = 3$ exceed those for $h = 2$, and the reverse is true for $\sigma_2/\sigma_1 > 1$. The interpretation for this is clearly the same as previously indicated for the discussion of Fig. 6.

Plots of f_3 vs. k are given in Fig. 10. Here as in Fig. 8 the current fractions for the case $h = 2$ are higher than those for $h = 3$. The reason for this has already been presented in the discussion of Fig. 8.

Before closing our treatment of intermediate strata in homogeneous earths we may note the analytical forms taken by the current fractions for the limiting case when the intermediate stratum is very thin. The first order terms of the expansion in $\Delta\bar{h}$ are:

$$\left.\begin{aligned} f_1 &= \frac{2}{\pi}\left[\tan^{-1}\frac{1}{\bar{a}} + \frac{2k\bar{a}\Delta\bar{h}}{1-k^2}\left\{\frac{1}{1+\bar{a}^2} - \frac{1}{9+\bar{a}^2}\right\}\right] \\ f_2 &= \frac{2}{\pi}\,\frac{1-k}{1+k}\,\frac{\bar{a}\Delta\bar{h}}{1+\bar{a}^2} \\ f_3 &= \frac{2}{\pi}\left[\tan^{-1}\bar{a} - \frac{\bar{a}\Delta\,\bar{h}}{1-k^2}\left\{\frac{1+k^2}{1+\bar{a}^2} - \frac{2k}{9+\bar{a}^2}\right\}\right], \end{aligned}\right\} \tag{41}$$

where $\Delta\bar{h}$ is the actual thickness of the intermediate stratum divided by the surface layer thickness h_1. In principle, these expressions should give the perturbing effect of the intermediate layer when $\Delta\bar{h}$ is very small. However, because of the character of the coefficients the approximation will also break down as k approaches the values ± 1. This, of course, is due to the fact that even though the intermediate layer may be of very small thickness it will nevertheless radically change the current distribution if its conductivity be made either very low or very high as compared to the rest of the conducting body.

Finally, the asymptotic variation for the current fractions in the general three-layer earth may be noted. These will be analogous to the expansions of Eq. (19), though we shall restrict ourselves now to the first terms only. These may be readily shown to be:

$$\left.\begin{aligned} f_1 &= \frac{2}{\pi\bar{a}}\,\frac{\sigma_1}{\sigma_3} \\ f_2 &= \frac{2(h-1)}{\pi\bar{a}}\,\frac{\sigma_2}{\sigma_3} \\ f_3 &= 1 - \frac{2}{\pi\bar{a}}\,\frac{\sigma_2}{\sigma_3}\left[\frac{\sigma_1}{\sigma_2} + h - 1\right]. \end{aligned}\right\} \tag{42}$$

These will clearly break down when σ_3 is small. As will be seen in a later section the current fractions will then assume exponential variations with $\bar{a}$, which can not, of course, be represented by the inverse power type of expansion.

DEEPEST LAYER OF ZERO CONDUCTIVITY; $k_3 = +1$

When the third layer has a very high resistance compared to the two upper strata, or in the limit when it actually has zero conductivity k_3 becomes unity. The denominator Δ in the integrals of Eqs. (23) then takes the form:

$$\Delta = 1 - e^{-2\alpha h} - k_2 e^{-2\alpha} + k_2 e^{-2\alpha(h-1)}.$$

Moreover f_3 will automatically vanish, so that, since the sum of f_1 and f_2 will be unity, only one of them need be calculated explicitly. We shall choose for convenience f_2, which will now be given by:

$$f_2 = \frac{4}{\pi}(1 - k_2)\int_0^\infty \frac{e^{-\alpha h}\sinh \alpha(h-1)\sin \alpha\bar{a}d\alpha}{\alpha\Delta}. \qquad (43)$$

Again, as in the type of conductivity stratification previously considered, the evaluation of the integrals giving the current fractions is much simpler when integral values of h are used. We shall therefore again restrict ourselves to integral values of h, and in particular to those where $h=2$, 3, or 4.

$h=2$

For this case Eq. (43) immediately reduces to:

$$f_2 = \frac{1-k_2}{\pi}\int_0^\infty \frac{\sin \alpha\bar{a}d\alpha}{\alpha\cosh\alpha} = \frac{1-k_2}{2}\left(1 - \frac{4}{\pi}\tan^{-1} e^{-\pi\bar{a}/2}\right), \qquad (44)$$

on recalling Eq. (18).

$h=3$

When $h=3$ Eq. (43) may be readily shown to reduce to:

$$f_2 = \frac{2(1-k_2)}{\pi}\int_0^\infty \frac{\cosh\alpha\sin\alpha\bar{a}d\alpha}{\alpha\left[\cosh 2\alpha + \frac{1-k_2}{2}\right]}. \qquad (45)$$

Both here and for $h=4$, as well as for the other general case, $k_3=-1$, which will be considered later, the integrals involved do not reduce to those given in standard tabulations. However, as the denominators of the integrals for these current fractions are the same as those occurring in the integral expressions for the potential distributions, a method which was developed in evaluating the latter[3] may also be applied here.

The application of that procedure shows that f_2 may be expressed as:

$$f_2 = \frac{2(1-k_2)}{3-k_2} - \frac{1-k_2}{2\sin\frac{\theta}{2}}B\left(\bar{a}, \frac{\theta}{2}\right), \qquad (46)$$

where $\theta=\cos^{-1}\frac{k_2-1}{2}$ and the function B is the rapidly converging series defined by:

$$B(\bar{a}, \theta) = \sum_{0}^{\infty} \frac{(-1)^n e^{-\bar{a}(n\pi+\theta)}}{n\pi + \theta} - \sum_{1}^{\infty} \frac{(-1)^n e^{-\bar{a}(n\pi-\theta)}}{n\pi - \theta}. \quad (47)$$

As previously indicated f_1 is, of course, equal to $1 - f_2$.

$h = 4$

Here f_2 may be first reduced to:

$$f_2 = \frac{1 - k_2}{2\pi} \int_0^{\infty} \frac{(4 \cosh^2 \alpha - 1) \sin \alpha \bar{a} d\alpha}{\alpha \cosh \alpha \left(\cosh 2\alpha - \frac{k_2}{2} \right)}. \quad (48)$$

The evaluation of the integral gives:

$$f_2 = \frac{3(1 - k_2)}{2(2 - k_2)} - \frac{1 - k_2}{2(2 + k_2)} \left[\frac{4}{\pi} \tan^{-1} e^{-\pi\bar{a}/2} + \frac{1 + k_2}{\sin \theta} B(\bar{a}, \theta) \right]. \quad (49)$$

where now $\theta = \frac{1}{2} \cos^{-1} k_2/2$.

For the special case where $k_2 = 0$, it may be readily shown that:

$$\begin{aligned} f_2 &= \frac{2}{\pi} \int_0^{\infty} \frac{\sin \alpha(h-1) \sin \alpha \bar{a} d\alpha}{\alpha \sinh \alpha h} \\ &= \frac{2}{\pi} \tan^{-1} \left(\tanh \frac{\pi \bar{a}}{2h} \tan \frac{\pi(h-1)}{2h} \right), \end{aligned} \quad (50)$$

which is valid for all values of h.

The curves of f_1 vs. $\bar{a}$ are plotted in Fig. 11 for $h = 2$, 3, and 4 and various values of the conductivity contrasts in the two upper layers. The highest dashed curves of the groups of three refer to $h = 2$, the middle solid curves correspond to $h = 3$, and the lowest dashed curves to those for $h = 4$. The general variation with a/h_1 and the conductivity contrasts are similar to those shown in the previous figures for f_1 vs. a/h_1. The decrease of the current fraction through the uppermost layer as the middle layer thickness increases is clearly due to the increasing current carrying power of the latter as its thickness increases relative to the surface stratum. It will also be noted that the curves in Fig. 11 are uniformly higher than those of Figs. 6 and 3 for the same thickness and conductivity parameters.

DEEPEST LAYER OF INFINITE CONDUCTIVITY; $k_3 = -1$

Here, too, we shall treat separately the individual cases where h takes on the integral values 2, 3, and 4. The only major change in the

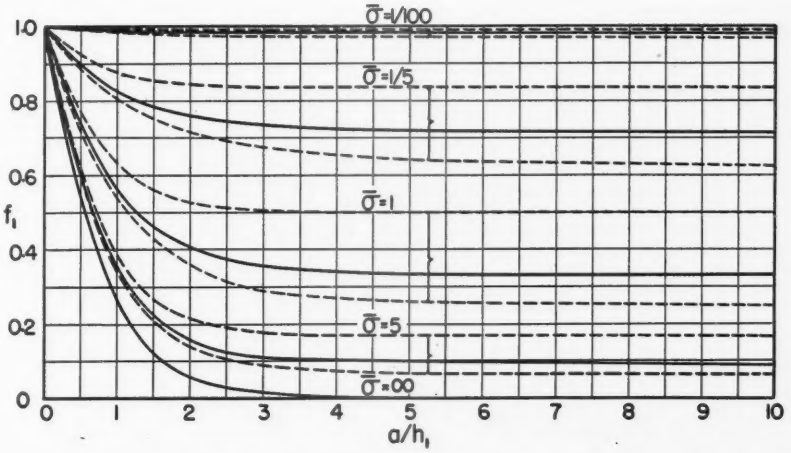

FIG. 11. Current fractions, f_1, in the surface stratum of a three-layer earth, the deepest of which has zero conductivity. Middle layer thickness = surface layer thickness, ($h=2$), for uppermost dashed curves; middle layer thickness = 2 · surface layer thickness, ($h=3$), for solid curves; middle layer thickness = 3 · surface layer thickness, ($h=4$), for lowest dashed curves. Rest of notation as in Fig. 4.

treatment for $k_3=-1$ with respect to $k_3=+1$ will lie in the fact that since f_3 will no longer be zero, two of the three penetration fractions will have to be calculated. Purely from the point of view of analytical convenience we shall choose for these the surface layer fraction f_1 and the third layer fraction f_3, which will be given respectively by:

$$\left.\begin{aligned} f_1 &= 1 - \frac{2(1-k_2)}{\pi}\int_0^\infty \frac{\cosh\alpha(h-1)\sin\alpha\bar{a}d\alpha}{\alpha[\cosh\alpha h - k_2\cosh\alpha(2-h)]} \\ f_3 &= \frac{2(1-k_2)}{\pi}\int_0^\infty \frac{\sin\alpha\bar{a}d\alpha}{\alpha[\cosh\alpha h - k_2\cosh\alpha(2-h)]}. \end{aligned}\right\} \quad (51)$$

The above expression for f_1 may be obtained either by a simple transformation of the f_1 of Eq. (23) or by subtracting from 1 the sum f_2+f_3. The results of the evaluation of these integrals for various values of h are as follows:

$h=2$

$$f_1 = \frac{(1-k_2)}{2\sin\theta}B(\bar{a},\theta), \quad (52)$$

where $\theta=\frac{1}{2}\cos^{-1}k_2$ for both f_1 and f_3.

$$f_3 = 1 - \frac{1 - k_2}{\sin 2\theta} A(\bar{a}, \theta), \tag{53}$$

where A is a series similar to B and is defined by:

$$A(\bar{a}, \theta) = \sum_0^\infty \frac{e^{-\bar{a}(n\pi+\theta)}}{n\pi + \theta} - \sum_1^\infty \frac{e^{-\bar{a}(n\pi-\theta)}}{n\pi - \theta}. \tag{54}$$

$h=3$

$$f_1 = \frac{1 - k_2}{3 + k_2}\left[\frac{4}{\pi}\tan^{-1} e^{-\pi\bar{a}/2} + \frac{\cos 2\theta}{\sin\theta} B(\bar{a}, \theta)\right]. \tag{55}$$

$$f_3 = 1 + \frac{1 - k_2}{3 + k_2}\left[\frac{4}{\pi}\tan^{-1} e^{-\pi\bar{a}/2} - \frac{1}{\sin\theta} B(\bar{a}, \theta)\right]. \tag{56}$$

with $\theta = \frac{1}{2}\cos^{-1}\frac{1+k_2}{2}$ for both f_1 and f_3.

$h=4$

For this case:

$$f_1 = \frac{1 - k_2}{\sqrt{k_2^2 + 8}}\left[\frac{\cos\frac{3\theta_1}{2}}{\sin\theta_1} B\left(\bar{a}, \frac{\theta_1}{2}\right) - \frac{\cos\frac{3\theta_2}{2}}{\sin\theta_2} B\left(\bar{a}, \frac{\theta_2}{2}\right)\right]. \tag{57}$$

and:

$$f_3 = 1 - \frac{1 - k_2}{\sqrt{k_2^2 + 8}}\left[\frac{A\left(\bar{a}, \frac{\theta_1}{2}\right)}{\sin\theta_1} - \frac{A\left(\bar{a}, \frac{\theta_2}{2}\right)}{\sin\theta_2}\right], \tag{58}$$

where:

$$\cos\theta_1 = \frac{k_2 + \sqrt{k_2^2 + 8}}{4}; \qquad \cos\theta_2 = \frac{k_2 - \sqrt{k_2^2 + 8}}{4}.$$

For the special case where $k_2=0$, f_1 and f_3 take the closed forms:

$$\left.\begin{aligned} f_1 &= 1 - \frac{2}{\pi}\int_0^\infty \frac{\cosh\alpha(h-1)\sin\alpha\bar{a}d\alpha}{\alpha\cosh\alpha h} \\ &= 1 - \frac{2}{\pi}\tan^{-1}\frac{\sinh\pi\bar{a}/2h}{\cos\pi(h-1)/2h} \\ f_3 &= \frac{2}{\pi}\int_0^\infty \frac{\sin\alpha\bar{a}d\alpha}{\alpha\cosh\alpha h} = 1 - \frac{4}{\pi}\tan^{-1} e^{-\pi\bar{a}/2h}. \end{aligned}\right\} \tag{59}$$

The curves of f_1 vs. $\bar{a}$ as calculated by means of the above equations are plotted in Fig. 12 for different conductivity contrasts of the two upper layers. Again the general trends are similar to those previously noted. The variation with the middle layer thickness, however, is reversed from that shown in Fig. 11 as here the current fraction in the upper layer increases with increasing middle layer thickness. This is clearly due to the fact that the shielding of the deepest conducting layer increases with the middle stratum thickness, thus decreasing the

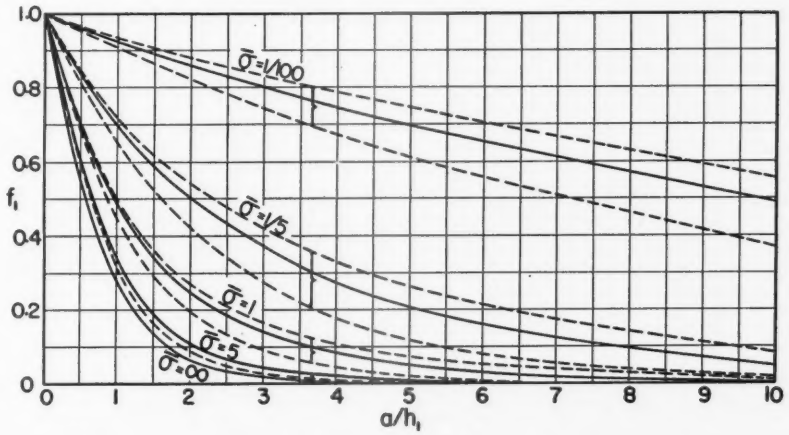

FIG. 12. Current fractions, f_1, in the surface stratum of a three-layer earth, the deepest of which has infinite conductivity. Middle layer thickness = surface layer thickness, ($h=2$), for lowest dashed curves; middle layer thickness = 2 · surface layer thickness, ($h=3$), for solid curves; middle layer thickness = 3 · surface layer thickness, ($h=4$) for uppermost dashed curves. Rest of notation as in Fig. 4.

current loss from the surface stratum. Here, as in all previous cases, the curve for $\sigma_2/\sigma_1=\infty$ is the same regardless of the middle layer thickness. The curve for $h=4$ and $\sigma_2/\sigma_1=5$ has been omitted because it lies so closely to that for $h=3$.

The corresponding current fractions in the middle layer are plotted in Fig. 13 for $h=3$ and $h=4$. Those for $h=2$ have been omitted because they would overlap too much those for the other cases. Here again it is to be noted that the current fractions for $h=4$ are higher than those for $h=3$ because of the increased current carrying capacity of the middle stratum with increased thickness. A new feature in these curves is the crossing of those for the same middle stratum thickness at larger values of a/h_1. That is, whereas for small a/h_1 the current frac-

tions decrease with decreasing middle stratum conductivity the reverse trend develops at large a/h_1. While this result must be accepted simply as a consequence of the numerical calculations it may be interpreted in the following way. At small values of a/h_1 the deepest layer does not appreciably come into play and the current distribution is divided between the two uppermost strata in a manner similar to that obtaining in the two-layer earth. For large values of a/h_1 the

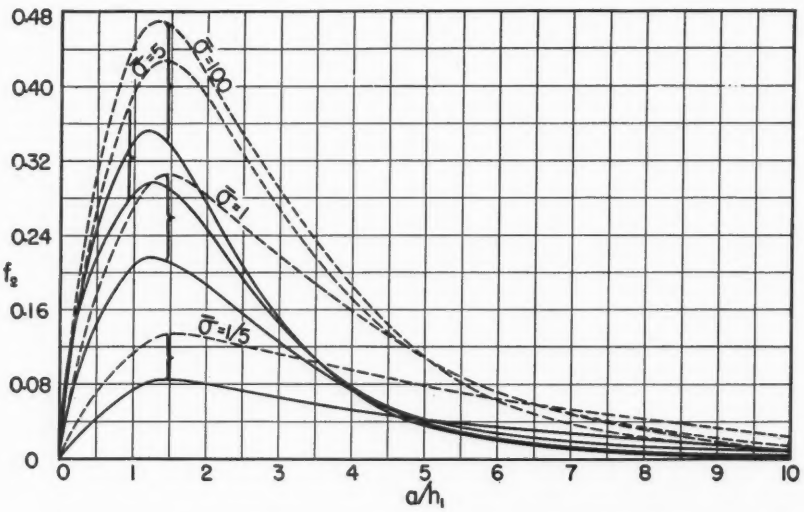

FIG. 13. Current fractions, f_2, in the middle stratum of a three-layer earth, the deepest of which has infinite conductivity. ---: middle layer thickness = 3 · surface layer thickness, ($h=4$); —: middle of layer thickness = 2 · surface layer thickness, ($h=3$). Rest of notation as in Fig. 4.

effect of the deepest layer can no longer be ignored. The current distribution between the second and third stratum then takes place in a manner similar to that in a two-layer earth. The effective electrode spacing for this composite system will be determined by the concentration or divergence of the current lines coming down from the surface stratum. Hence if the middle layer conductivity is made relatively high compared to that of the surface stratum it will draw down the current from the latter in an almost vertical direction and place the effective sources in the second stratum at separations which are almost as great as the actual electrode spread on the surface. For such spreads the middle stratum will, of course, lose most of its current to the deep-

est layer. If, however, the middle layer conductivity is relatively low the current entering it from the surface stratum will be highly diffused and the effective spread of the virtual electrodes in the second layer will be considerably reduced. Under such conditions the middle layer will retain a greater fraction of its current and, depending upon the

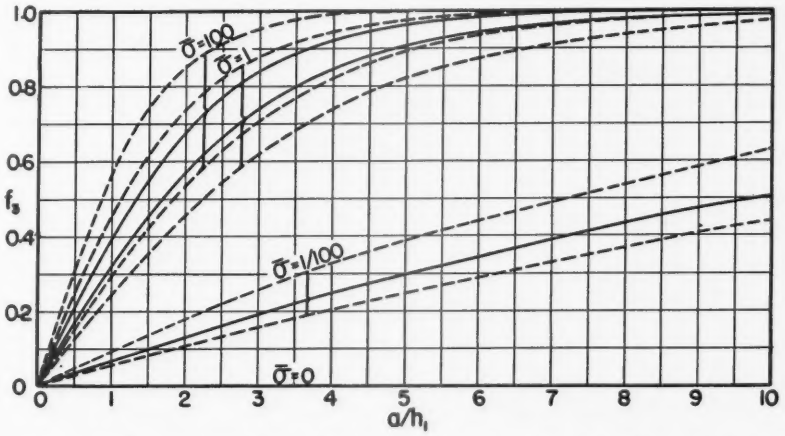

FIG. 14. Current fractions, f_3, in the deepest stratum of a three-layer earth, this stratum having infinite conductivity. Middle layer thickness = surface layer thickness, ($h=2$), for uppermost dashed curves; middle layer thickness = 2 · surface layer thickness, ($h=3$), for solid curves; middle layer thickness = 3 · surface layer thickness, ($h=4$), for lowest dashed curves. Rest of notation as in Fig. 4.

exact balance of the numerical magnitudes, may ultimately carry even more current than when the second layer conductivity is high.

The current fractions in the deepest stratum are plotted in Fig. 14. Again the shielding effects of the middle stratum are shown by the fall in the current fraction f_3 as the middle layer thickness is increased.

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PROSPECTING EFFECTIVENESS*

ESME EUGENE ROSAIRE†

ABSTRACT

The overall prospecting effectiveness or power of any given method is made up of four component parts. Three of these tend to increase with successful use and the passage of time. These are (a), the *operating effectiveness*, which, referring to getting the data, is based on the cost of field operations in time and money; (b), the *technical effectiveness*, which, referring to the quality of the data, is measured by the magnitude of the observational errors; and, (c), the *resolving effectiveness*, which, referring to the interpretation of the data, is measured by the ability to translate the data into terms of economic significance. The fourth component tends to decrease with successful use and the passage of time, and is, (d), the *finding effectiveness*, which is determined by the ratio of the magnitudes of findable anomalies to the magnitudes of the current observational errors. At any given time, the overall prospecting effectiveness of a given method is determined by that particular component which displays the minimum effectiveness.

In the major petroleum producing provinces of the U.S.A., negligible or decreasing *structure* finding effectiveness is displayed by all of the successfully used structural prospecting methods with one exception, the drill, the overall structural prospecting power of which is limited by a low operating effectiveness. However, these methods with existing low structural finding powers, surface geology, gravity methods, and seismic methods, display a more than appreciable finding effectiveness for anomalies which cannot be readily interpreted in terms of structure. This situation indicates that the existing low overall efficacy of these various methods is determined not so much by a low absolute finding effectiveness as by a resolving power which is low because of structural inhibitions upon interpretation.

The sedimentary hydrocarbon prospecting method was introduced upon a theoretical basis which involved the depth of the source and the postulate of local lateral homogeneity, but did not attain any measure of overall prospecting potency until its interpretation technique was relieved of such purely theoretical inhibitions. Subsequent development has shown that similar limitations upon refraction interpretation were responsible for the collapse of that method in 1930 after the method had displayed marked prospecting effectiveness for shallow high relief structure. Attention is called to the possibility that the removal of similar inhibitions also may result in a similar increase in resolving power for other methods which today display a low structure finding effectiveness, and thus lead to their renaissance because of their thereby increased overall prospecting effectiveness.

In the past, several terms have been used to describe the general prospecting effectiveness of any given exploration technique, such as its "resolving" or "finding" power. An element of confusion has been present in discussion, since the technical excellence of a method may improve by use and yet, for all practical purposes, that same vastly improved technique may approach obsolescence.

In a recent historical study which this writer has made of refraction prospecting, it became apparent that several factors were present, each of which, at one time or another, determined the overall prospect-

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ing effectiveness of that method. Desirable clarification followed from the recognition of these several factors, and, since they appear to be generally present in the operation of other prospecting methods, they are outlined below, briefly described, and illustrated by reference to pertinent instances of the past.

OPERATING EFFECTIVENESS

The *operating* effectiveness refers primarily to the field effectiveness, that is, to the cost of getting the data under the varying conditions prevailing throughout the potential theatre of operations.*

About 1925 when the refraction method was established through confirmation by discovery, its operating effectiveness was sufficient for the immediate problem at hand, for there was then an appreciable unexplored area in the Gulf Coast where the originally introduced Seismos technique was operative at a not unreasonable cost. In 1926, however, the American-introduced techniques materially increased the immediate as well as the potential operating effectiveness of the method.

The immediate increase in operating effectiveness came about through the use of radio transmission of the time break and the reliance upon the air wave for distance determination. This afforded some reduction in the cost of obtaining the data through the use of buried charges which cut down the cost of damage claims and the elimination of survey crews. The potential increase in operating effectiveness followed from the use of electrically recording seismographs which made it possible, a year later, to operate at reasonable costs over marshy and water covered areas, thus extending materially the operable theatre of operations. In addition, the longer shot recorder distances, then possible, extended materially the operable theatre of operations in the third dimension.

In the middle thirties the Salt Dome Oil Company extended the

* The potential theatre of operations includes all those areas of the earth's surface where the sedimentary conditions, i.e., the geology, are favorable to the accumulation of petroleum, and obviously, is three dimensional in nature. It is apparent that any given organization at any particular time, voluntarily or involuntarily, may direct its attention to only part of the potential theatre of operations. Proximity to markets, wars, company or national policies, or individual preferences also may limit the effective theatre of operations for any given organization, but from the standpoint of a given method, its potential operating effectiveness eventually must be considered with respect to its use now or at some time in the future throughout the whole potential theatre of operations to which there are only geological boundaries.

operating efficiency of the torsion balance into water-covered areas by the use of piling and barges for station locations in the shallow waters of Galveston Bay, Texas. The development of field gravity meters increased tremendously the operating effectiveness of gravity methods because of the much shorter time required to secure data. The development of submarine gravity meters bids fair to result in an increase in the operating effectiveness of gravity prospecting, again through an extension of the operable theatre of operations.

The introduction of surface seismometers increased the operating effectiveness for reflections, in simplifying recording as the number of seismometers increased.

Operating effectiveness is measured in terms of money and time, for these are the two primary expendables in prospecting.

From the standpoint of operating effectiveness, the passage of time is on the side of the operator, in that it is reasonable to expect that progress will come with increased experience, and therefore, that the operating effectiveness of a method will be higher at a later date.

TECHNICAL EFFECTIVENESS

The *technical* effectiveness or power of a method refers to the quality of the resulting data, i.e., to the observational errors.

The originally introduced Seismos refraction method relied upon the use of surveyed distances and the calculation of the shot instant back from the air-wave arrival instant (using an air travel speed calculated from measurements made of azimuth, air temperatures, wind directions and velocities). Appreciable observational errors were inherent in that early practice, and so the timed intervals on the record were one-fifth of a second. At shot-to-recorder distances much above two miles the observational errors increased tremendously, thus limiting the coverage and penetration.

The American-introduced innovation of transmitting the shot instant by radio increased the technical effectiveness of the refraction method by reducing the observational error in the determination of the travel time of the seismic impulse. This justified reducing the timed intervals on the record from one-fifth to one-fiftieth of a second, and permitted the increase of the shot-to-recorder distance from two to more than six miles, with a corresponding material increase in penetration.

The development of torsion wires with lowered temperature coefficients of twist caused a marked increase in the efficacy of the torsion

balance. A similar increase in the technical effectiveness of magnetic prospecting followed from the introduction of beams automatically corrected for temperature variations, and also from the use of photographically recording base-station magnetometers.

Least square methods of "smoothing" the original data represent attempts to increase the technical effectiveness of the method in question.

From the standpoint of technical effectiveness, the passage of time again is on the side of the operator, in that here also, it is reasonable to expect that progress will come with increased experience, and therefore, that the technical effectiveness of a method will be higher at a later date.

RESOLVING EFFECTIVENESS

The *resolving* effectiveness of a given method is the extent to which the data can be translated into terms of economic significance. Thus, resolving effectiveness deals with the interpretation or the understanding of the data.

A novice in surface geology may report dips whose magnitudes are several times the magnitude of his observational errors, but his chief, with better "understanding," may rule out most of these as nonsignificant because they are due to cross bedding or to surface slumping, thus displaying his higher personal resolving effectiveness.

Gravity meter anomalies are said to be less closely related than torsion balance anomalies to "shallow gravel beds" of no economic significance, and, to the extent that this postulate is correct, the gravity meter can be said to have increased the resolving power of gravity prospecting.

Gravity prospecting was first introduced into the Gulf Coast with gravity maxima as objectives. This followed from the early work at Spindletop, which was displayed as a well defined gravity "high." The discovery of the Nash, Clemens, and Allen salt domes confirmed that early interpretation or "understanding" of the data, but two years marked by failures, followed before the discovery of Sugarland extended the resolving power of the method by setting up gravity minima as the more generally favorable objectives. That change in objectives marked a tremendous increase in the resolving effectiveness of gravity prospecting.

Well velocity determinations increased the resolving effectiveness of reflections by improving the translation of reflection travel times into actual depths, thereby achieving greater economic significance.

Electrical logging led to a material increase in the resolving effectiveness of subsurface stratigraphic studies.

Recognition of the halo pattern was a major step in increasing the resolving effectiveness of free soil-gas and sedimentary hydrocarbon prospecting methods, as also was the recognition of hydrocarbon leakage above the point at which a fault terminates petroleum accumulation. Further, the analysis of soil samples for hydrocarbons of significant origin rather than for the multitudinous constituents of unknown but probably secondary origin, results in a materially increased resolving effectiveness.

Time is also on the side of resolving effectiveness, for in our interpretation of data we always must venture postulates about the unknown with less data and information today than we will have as further development takes place.

These three components, operating, technical, and resolving capabilities, all tend to *increase* with the passage of time, and if they alone comprised the prospecting effectiveness of a method, each year of application would result in the better recognition of more and more objectives. But, as we well know, that condition does not exist. Obviously, there remains one additional component which tends to *decrease* with continued successful use.

FINDING EFFECTIVENESS

The *finding* effectiveness is the ratio, at any given time, of the general magnitudes of the findable anomalies of possible economic significance to the general magnitudes of the current observational errors.

The recognition, about 1927–1928, that better reflection records followed from shooting below the water table resulted in a material increase in the finding effectiveness of that method.

But, though time can be expected to bring increased operating, technical, and resolving effectiveness, the reverse is true for finding effectiveness. And since we exploit natural mineral resources at a much higher rate than that at which they are formed in nature, continued successful application reduces the finding power not only of the method used, but also of any other method with a similar objective. For example, since surface geological methods had been used successfully and intensively in the U.S.A., aerial photography never displayed a very high overall prospecting value in spite of high operating, technical, and resolving powers.

It is apparent that the concentration of effort by a given method in one particular localized theatre of operations eventually will reduce its finding effectiveness to the point of diminishing returns. If the method then is applied to another particular theatre of operations, new to that method, its finding power may be raised above the point of diminishing returns. Thus in 1935–1936, when the reflection method was approaching its peak effectiveness in the Gulf Coast, it was introduced into the Illinois Basin where it had not been used previously, and a marked increase in the finding effectiveness of the method followed from its introduction into an area new to the method.

The potential finding power of any given method cannot be considered as having reached the point of diminishing returns until the potential theatre of operations has been worked in detail by that method. However, since any particular organization rarely is interested in prospecting throughout the potential theatre of operations, we often see that for all practical purposes the actual finding effectiveness of a method has vanished long before its world-wide use has been attained.

At first glance, these classifications may not appear clean cut, for in some cases, they are not too readily differentiated. However, these classifications do have some merit in clarification, as further study will show.

For instance, instead of considering effectiveness let us consider the limits upon effectiveness. All of us are familiar with the term "observational error." For "error," let us substitute the word "hazard," and then speak of the overall prospecting hazard and its component parts, the operating, technical, resolving, and finding hazards.

We can recognize the tremendous structural finding hazard (or, if one prefers, the non-uniformity in finding power) of the pre-geophysical Gulf Coast prospecting methods in their failure to find the Hawkinsville salt dome, with cap rock only 95 feet below the surface of the ground, whereas they did find several other salt domes with cap rock or salt at depths more than ten times that later found at Hawkinsville.

In the transition from the refraction to the reflection method, the operating hazard changed materially. In refractions, large charges had been exploded at infrequent intervals, with a resulting hazard which was primarily one of public liability. In reflections, however, smaller charges were used, materially reducing the public and property hazards. However, the greater frequency at which the charges were fired increased the operating hazard for the shooter and his assistants.

for there always is a tendency for carelessness to develop in the case of routine operations. As events proved, the transition from refractions to reflections resulted in the shift of an important operating hazard from the public to the field crews.

Just as the strength of a chain is only that of its weakest link, and as the speed of a convoy is that of its slowest unit, just so the prospecting effectiveness of a given method is established by the lowest effectiveness of any one of these four components.

As has been pointed out, continued use tends to raise the operating, technical, and resolving powers of a given method. On the other hand, continued successful use tends to decrease the finding power of the method, so that eventually the prospecting effectiveness of a given method decreases with time, i.e., becomes that of the lowest component power, the finding effectiveness. So, with continued successful use, any prospecting method may be expected to approach obsolescence.

As a general rule the structures of highest relief are fewest in number, and the number of structures tends to increase as the considered relief decreases. The same is true of petroleum accumulations, for there are many more thin sand fields than there are thick sand fields like Kettleman Hills and Conroe. Thus, whether the method be indirect or direct, the number of anomalies tends to increase as we approach those of marginal magnitude.

A reasonably general history of a prospecting method should, then, be about as follows. At the time of its introduction its operating, technical, and resolving powers are at a minimum. In contrast, however, the finding power, if not then at a maximum, peaks at or shortly after the time of successful reduction to practice. The activity curve climbs, slowly at first, and then more and more rapidly as confirmations by discovery follow. And, although each success in turn decreases the finding power of the method, each improvement in technical effectiveness results in a lowered observational error, thus making available a greater number of findable anomalies of a lower level of magnitude.

Conceivably, it is possible to set up conditions where an increase in exploration activity more than offsets the increased number of marginal anomalies, leading to a rapid decline in finding effectiveness. However, since such conditions are rarely attained in practice, the finding effectiveness should tend to decrease gradually with time. In those cases where the finding effectiveness was the limiting factor, the overall prospecting effectiveness also would decline gradually from a

maximum. Thus, when the method approaches "normal" obsolescence, the finding effectiveness is approaching zero asymptotically, while its operating, technical, and resolving powers have been developed materially and still may be increasing.

That anticipated historical development did not occur in the case of refractions, for the overall prospecting effectiveness of that method declined suddenly in 1930, at a time when the method still was finding a more or less adequate supply of anomalies definitely greater in magnitude than the then current observational errors.* That collapse could hardly have been due to inadequate operating or technical powers, for these had shown a quite satisfactory improvement with time. If, as the evidence shows, the operating, technical, and finding powers of the refraction seismograph were adequate in 1930, its collapse in that year can have been due only to current inadequacy of the remaining component, the resolving effectiveness. This conclusion is borne out by subsequent developments,[1] which show that, since a surprisingly high percentage of these refraction dry holes bear a typical relationship to subsequently discovered or extended production, they must be considered as close misses rather than as outright failures, i.e., due to the erroneous interpretation of potentially significant data.

From these considerations, it becomes evident that the prospecting effectiveness of the refraction method ultimately was limited by the *resolving* power of the method as of 1930, not, as has generally been considered to be the case, to its having reached its potential prospecting effectiveness. Here then, we have an illustration of the importance of resolution hazards which are analogous to the technical hazards, or, more familiarly, the observational errors.

With these components of prospecting effectiveness in mind, we can recognize their interplay as follows in the various forms of prospecting used successfully to date.

SURFACE GEOLOGY

The first structural prospecting tactic was that of mapping structure exposed at the surface of the earth. The operating effectiveness was high, in that the cost of surface inspection by a competent geolo-

* In 1930, there were twenty-two wells drilled on refraction anomalies, of which seventeen were charged off as failures.

[1] E. E. Rosaire. "An Analysis of the Refraction Collapse of 1930." Presented at the Annual Meeting, Houston, Texas, April 1, 1941.

gist was not at all expensive. We can recognize a major increase in resolving effectiveness with the introduction of contour maps.

A reduction in observational errors, i.e., an increase in the technical effectiveness, was achieved by the use of various methods of running levels. First came devices like the Brunton compass and odometer, and then the more familiar plane table and alidade. The corresponding increase in technical effectiveness was achieved at a slight but not prohibitive decrease in operating effectiveness, for the three man crew (plane table man, rodman, and chief of party) increased the monthly cost about three times, and certainly increased the cost per acre examined. Eventually, however, the supply of exposed but untested structures and noses was exhausted, and so surface prospecting became obsolete when its structure finding effectiveness showed only marginal returns.

However, there remain an appreciable number of anomalies which often are obvious in surface studies. These are physiographic in nature, and have been disregarded if they have no obvious structural significance. Many of these are of the class undignified by the term "creekology," and, as anomalies, frequently display magnitudes much greater than the observational errors of surface geological mapping. Later development often shows that they have a striking, if unexplained, relationship to petroleum accumulation. The existence of such findable surface anomalies indicates that, today, surface geological prospecting is limited by its resolving effectiveness.

SUBSURFACE PROSPECTING

As the structural prospecting effectiveness of surface methods waned, subsurface prospecting was introduced. The first step was pit digging or trenching to make artificial exposures. That trend continued, first in the form of core drilling, and later in the form of subsurface studies as we know them today. The introduction of micropaleontological methods marked a tremendous increase in the resolving effectiveness of subsurface prospecting, as did the introduction of electrical well logging.

Since all the petroleum in the world could be discovered by drilling enough wells to the basement complex, subsurface prospecting has the highest finding power of all possible methods, for a bore hole can yield not only structural data, but provides the test which eventually is required to establish the actual discovery of petroleum. Characterized also by data with low observational errors and almost obvious transla-

tion into terms of structural relationship, though very high in cost, the drill is a method which can be described as one with the highest possible technical, resolving, and finding powers, but one whose overall prospecting effectiveness is limited by an inordinately low operating effectiveness.

About 1920, prospecting for petroleum was limited entirely to the direct structural approach, at a time when electrical logging had not been introduced, when micropaleontological methods were just being instituted, and when the depths of drilling were much shallower than they are today.

In his war against Nature, the prospector had only two types of fire power. One was surface structural prospecting, which, though limited in penetration and range, had proved to be quite adequate in the preliminary but quite important task of clearing away the defending screen and isolating the strong points of Nature, the exposed structural features. The second weapon was subsurface structural prospecting, relying upon the drill, the large caliber siege gun. That "Big Bertha," with maximum range and penetration, then, as it is today, was the proper equipment for the final reduction of the isolated strong points, but was, as it is today, too costly and too unwieldy for the routine of preliminary skirmishing.

About 1920–1923, the lighter weapon, surface prospecting, had performed yeoman duty, but was proving too small in caliber to operate against the second line of defense (buried structure) to which the forces of Nature had been driven back. While "Big Bertha" still could function, its routine use was too much like using "battle wagons" to hunt down submarines. That is, the ultimate successful outcome of such a hunt would be inevitable, but only at tremendous costs and with inordinate hazards.

That was the period when a great outcry was raised as to the inadequate magnitude of our petroleum reserves, leading to the acquisition of shale oil leases and the Bergius process for the hydrogenation of coal, and to the contemplated use of colloidal graphite and vegetable oils for lubrication purposes.

The solution did not lie in the substitution of these new strategies of *ersatz* for the established strategy of prospecting for petroleum, nor in continued reliance upon a "Big Bertha" of improved performance but of just as limited operating effectiveness. Rather, the solution lay in the development of new light weapons with improved "fire power," but with an operating effectiveness adequate for the particular prob-

lems involved in the *then* current task of strong point isolation. That solution took the form of the introduction of geophysical methods which effected a renaissance of the basic structural approach. The torsion balance and the refraction seismograph were the first of these to appear on the scene and to display adequate prospecting effectiveness.

THE TORSION BALANCE

This, the first of the gravity devices, was introduced into the Gulf Coast as a purely structural device, and was tried out first at Spindletop, which was the first of the many Holy Grails of Gulf Coast prospecting.* An outstanding gravity maximum was recognized there, which set the pattern for further Gulf Coast objectives. Success followed at Nash, Clemens, and Allen, but soon the dry holes charged against the method caught up with and surpassed the credits noted above. The passage of time brought bettered field methods and lowered observational errors, but, although an appreciable number of gravity anomalies were still being found, the method was being relegated inevitably to obsolescence. In spite of these improved operating and technical powers and adequately high finding powers, the overall prospecting effectiveness was limited by the inadequate resolving power of the method.

The discovery of oil at Sugarland, however, focussed attention upon the importance of a new objective, the gravity minimum, and the use of the method increased materially. Note, however, that this increase in overall prospecting effectiveness followed solely from a demonstrated increase in resolving effectiveness.

Increased activity in gravity prospecting led to the consideration of other gravity devices.

Next to appear on the scene was the pendulum, from which, apparently, there was a hope of increased technical effectiveness due to the possibility of securing actual rather than relative data. Although one discovery, Cleveland, is credited to this gravity method, and perhaps others not publicly known, its low operating effectiveness led to a short life.

The third gravity method to appear on the scene was the gravity meter. In contrast to the low operating effectiveness of the pendulum, the outstanding characteristic of the gravity meter was a very high operating effectiveness; a per station cost small in time as well as

* Others were Humble, Sugarland, Van, Thompsons, Conroe, Iowa, Hastings, Friendswood, LaFitte, Eola, Paradis, Hawkins, etc.

money when compared to the per station cost of either the torsion balance or the pendulum. Obviously, in the Gulf Coast theatre of operations, the gravity meter had little to contribute except to new entrants, who thereby were able to set up gravity libraries comparable in quality at much lower costs to those which their predecessors had secured the harder way, i.e., with the torsion balance. So, in the Gulf Coast theatre of operations, the gravity meter was confronted with much the same situation as that which confronted aerial mapping. The overall prospecting effectiveness was limited by a finding power reduced by the previous use of a method of admittedly lower operating effectiveness, and was not to be retrieved by a belated if material increase in operating effectiveness.

Today we see the following situation in the Gulf Coast threatre of operations, where gravity prospecting undoubtedly has scored its outstanding successes. Major compaigners, old and new, have practically complete files of gravity coverage, and many minor campaigners have less complete but still quite adequate files. All told, these data represent staggering total acquisition costs. These data have been studied and restudied to various degrees, involving even the use of electrically operated computing devices for least squaring the data. And, after all the manhandling to improve the technical effectiveness of the data, the lowered observational errors have not increased materially the overall prospecting effectiveness, for unexplained gravity anomalies still exist greater in magnitude than the associated observational errors.*

Obviously, the overall prospecting effectiveness of gravity prospecting is limited today by its low resolving power. This same situation, it will be recalled, existed once before in the same theatre of operations, in 1925–1926, when the overall prospecting effectiveness of the gravity method was reestablished simply by an increase in resolving power.

THE REFRACTION SEISMOGRAPH

This method was introduced over the period 1920–1924, during which time only indifferent success followed in several theatres of

* A case in point is the recent discovery at Hawkins in Wood County, Texas. There the primary credit for confirmation goes to core drilling, in spite of the fact that, some time before that method of very low operating effectiveness was used, a gravity anomaly of appreciable magnitude had been found in the immediate area by the torsion balance campaign there circa 1928–1930. The observed gravity minimum had been discounted because it was thought to be the manifestation of a regional syncline.

operations, Poland, Mexico, the Mid-Continent, the Mexia-Powell Fault Line, and the Gulf Coast. In all these early trials, the method used was that of refraction profiles from which depths were calculated to seismic horizons identified by seismic travel speeds.

In that form the method displayed a low operating effectiveness, for profiling involves a high per-acre cost. At the time, the technical effectiveness was not inadequate, and in general, the finding effectiveness was fairly high.

In Mr. L. P. Garrett's initiation of fan tactics, we can recognize a major increase in operating effectiveness, leading to a material reduction in the cost per acre surveyed. Coupled with an ideal opportunity in the form of the many shallow salt domes undiscovered by the prospecting methods previously used in this area, many confirmations by discovery followed. Here we see a case where a material increase in the operating effectiveness of a method resulted in a corresponding material increase in its overall prospecting effectiveness.

However, a sequence of events followed which were misinterpreted at the time. Shortly after successful reduction to practice, the refraction method suddenly showed an alarmingly high percentage of failures in 1926. Major refinements in the method were introduced by the American technicians, which resulted in a major increase in the overall prospecting efficacy of the method. At the time, those innovations were given full and entire credit not only for the overall increased operating effectiveness, but also for the improvement required to bring the prospecting effectiveness back to its earlier level. This idea was so firmly engrained in the operators' minds at the time that somewhat later the method was used extensively in exactly the same way, i.e., refraction profiling for depth calculations, as it first had been tried with only indifferent success. With the passage of time, the veil has cleared enough to show that, for similar terrain, the refraction profiling of 1928–1930 was more successful than the refraction profiling of 1923–1924 only to the extent of the increase in operating and technical effectiveness which had taken place in the interim. As subsequent development has shown, these improvements were not enough to justify the expenditure involved in the refraction profile campaigns in the Permian Basin from 1928–1930. Data of equivalent significance could have been obtained at much lower costs had the increased operating effectiveness due to fan shooting been utilized, a step which had been taken before the introduction of the refinements originated by the American technicians.

Reverting to refraction operations in the Gulf Coast theatre of operations, the prospecting effectiveness of the refraction method collapsed in 1930, when the drill showed seventeen failures out of twenty-two trials. Since these dry holes were drilled on refraction anomalies which were appreciably greater in magnitude than the observational errors involved, the finding effectiveness of the method was adequately high at the time. Obviously, the operating and the technical powers also were adequate, or the method would not then have reached its peak activity. Subsequent developments show that more than half of the refraction dry holes bear a striking and typical relationship to subsequently discovered or extended production. Interestingly enough, to date this same relationship has been found to hold relatively better for the early refraction failures than for the later refraction failures using the American introduced improvements, though this relative difference may decrease with time.

We are now in position to recognize that the American introduced improvements in refraction prospecting increased the overall prospecting effectiveness of the method by virtue of increased operating and technical efficacies. However, we now recognize that the early refraction failures took place because of limitations upon the resolving power of the method. And, further, since the 1930 failures fall into the same class as the earlier refraction failures, we also are forced to recognize that the American introduced improvements did not increase the resolving effectiveness of the refraction method. Here too, we see a case where the overall structural prospecting effectiveness of a method was determined by the current limitations upon its resolving power.

THE REFLECTION SEISMOGRAPH

This method also was introduced as a purely structural method, for it was recognized early that reflections approached more closely than refractions to that direct structural prospecting method, the drill.

When compared with refraction fans, the reflection method showed a much higher per-acre cost, and so was characterized by a lower operating effectiveness. However, a material increase in the overall prospecting effectiveness of reflections followed from two factors. The first was an increased technical effectiveness in that lowered (structural) observational errors were achieved. The second was an increased finding power in that refractions could map only underlying beds of increasing seismic travel speeds whereas reflections enjoyed no such limitation.

When compared with the direct structural method, the drill, we recognize several limitations to reflections. One of these is the inability to recognize faults of appreciable throw, except under favorable conditions. Another is the inability to "call the shot," in that the desired structural objective (i.e., horizon) is not always an adequate reflecting horizon. Still another is the difficulty of establishing the proper equivalent seismic travel speeds required to translate the reflection travel times into actual depths. All of these factors are definite and critical limitations upon the finding effectiveness of reflections.

As would be expected of a method which most closely approximates that direct structural method, the drill, the reflection method has done yeoman duty, with a brilliant record of confirmation in the Mid-Continent, the Gulf Coast, East Texas, California, Illinois, and elsewhere. Following in the wake of high-grading by at least four structural prospecting methods,* it may well be that reflections will prove to have displayed the greatest overall prospecting effectiveness of all *structural* prospecting methods. But today, reflections show marginal prospecting effectiveness not only in the five above named major producing provinces, but also in a province of potential but as yet unproved production importance, central Mississippi.†

* Surface geology, the torsion balance, refractions, and the ubiquitous drill.

† The reflection campaign of 1939–1940 in Central Mississippi was touched off by the discovery of oil at Tinsley, September 5, 1939. Since that time only one other discovery has followed, the four-well Pickens oil field, although the operating activity reached a reported maximum of forty crews.

The finding power of the reflection seismograph in this campaign appears to have been adequate, for an appreciable number of reflection prospects have been tested by the drill. The results have been disappointing, however, in that the predicted structural relief has not been confirmed by the drill in many cases.

Earlier (1935–1938), the reflection seismograph had been introduced into the Illinois Basin, and there was markedly successful from the start. In that passage of time, the operating and technical effectiveness of the method certainly did not decrease, and has been reported to have increased. Consequently, the failure of the recent reflection campaign in Mississippi very probably represents an existing limitation upon the resolving effectiveness of the method; an inability to translate the Mississippi reflection data into terms of economic significance (buried structure).

This limitation upon the method's resolving effectiveness very probably will be found to be of the same nature as that which led to the collapse of refractions in 1930; the failure to take into account local changes in seismic travel speeds due to the presence of annular geochemical anomalies.

Equivalent geochemical changes would affect the seismic travel speeds proportionately less in the Illinois column of older geological sediments than in the Mississippi column of younger geological sediments, for the normal seismic travel speeds generally increase as the geological age increases for a particular type of sediment.

In spite of the brilliant record of reflection achievement, it is hardly "lese majesty" if we look for the cause of its declining prospecting effectiveness, for that trait has been common to all previously and successfully used structural methods. Admittedly, and as would be expected, its finding effectiveness has decreased, for the record of achievement is impressive. On the other hand, low as the finding effectiveness is today, there are a number of reflection structures which, when being tested by the drill, are found to be spurious or uncomfortably close thereto. True, the magnitudes of these spurious reflection structures often are close to the observational errors of the method, but, on the other hand, there are a growing number of cases where the apex of the reflection structure is found later to be displaced from the actual structural apex.

Thus, there is definite evidence pointing to the possibility that the finding effectiveness of reflections has not yet become marginal, and that as in the case of other structural methods, the prospecting effectiveness of the method may be limited today by an inadequate resolving effectiveness.

To date then, the prospecting industry has placed organized reliance upon five structural methods. Of these, only one, the drill, still possesses adequate structural finding power, but its overall prospecting effectiveness is limited by its low operating effectiveness—its high expense in cost and time. The *structural* prospecting effectiveness of the four others, surface geology, gravity methods, refractions, and reflections, are limited today in the major petroleum producing provinces of the U.S.A., by negligible or nearly negligible *structural* finding powers.

GEOCHEMICAL PROSPECTING

As it is taking form today, geochemical prospecting is defined to include all prospecting methods which have as objectives the recognition and localization of either the petroleum accumulation itself or the manifestations of the petroleum accumulation in its immediate sedimentary environment. As so defined, geochemical prospecting is a direct approach as contrasted to the earlier indirect approach of structural prospecting. Though linked genetically to the earlier and very successful approach of searching for visible seeps of oil and gas, a chronological gap of about one generation intervened between the two basic direct prospecting methods of macro and micro geochemical prospecting.

FREE SOIL-GAS PROSPECTING

The first rational step towards micro geochemical prospecting (as defined above) appears to have been the collection of samples of free gas in the near-surface soil, and the analysis of these gas samples for traces of saturated hydrocarbons.[2] This technique was introduced about 1930, and has been under investigation since that time. The greater part of the early literature is in Russian, for the Russian investigators appear to have given that technique more literary if not more general support than investigators of other countries.

In the first steps towards developing the resolving effectiveness of this method the approach was based entirely upon considerations of depth and local lateral homogeneity, in that the postulate was made that the center of the free soil-gas anomaly should coincide with the center of the trap associated with the petroleum accumulation to which the anomaly owed its origin.[3] Formulae were developed for calculating the depth of a petroleum accumulation which were based upon the shapes of free-soil-gas anomalies resulting from assumed leakage characteristics.

With regard to the failure of the free soil-gas method to attain an initial overall prospecting effectiveness, the following analysis can be made.

Due to the difficulty of developing field analytical equipment, the method was based upon the summertime collection of free soil-gas samples which were transported to a central laboratory for analysis during the winter. The reduction in analytical costs through the use of a central laboratory was more than offset by the appreciable time lag which intervened between the sample collection and sample analysis. As a result, the net operating effectiveness was low.

The sample collection was based upon the extraction of free soil-gas by the use of a pump operating upon a shallow bore hole. The Russian investigators called attention to the limitations of that process, particularly in the case of permeable soils, when the sample usually consisted largely of atmospheric air. On the other hand, the above mentioned technique obviously would not permit the collection of gas samples below the water table. Further, the absence of a satisfactory routine technique for separating methane from the saturated hydrocarbons of higher molecular weight (presumably with a unique origin

[2] G. Laubmeyer. U. S. Patent No. 1,843,878, issued Feb. 2, 1932.

[3] P. L. Antonov. "Contribution to the Theory of the Gas Survey," Neftyanoye Khozyaystvo, Vol. 26, No. 5, pp. 19–23, 1934.

in a petroleum accumulation) resulted in the method relying to an undue extent upon the use of methane as a criterion. These limitations resulted in a low technical effectiveness.

However, in spite of these limited operating and technical effectivenesses the finding power appears to have been above marginal, for the early data of record show several anomalies with magnitudes greater than the apparent observational errors. Even so, there is no outstanding evidence which points to any appreciable number of cases where the soil-gas method was credited with confirmation by drilling.

Here we see a method which displayed some measure of finding power in spite of low operating and technical powers, but which showed a negligible overall prospecting effectiveness.

SEDIMENTARY HYDROCARBON PROSPECTING

Such success as this potentially direct method has attained has followed from an improvement of major importance—the analyses of soils rather than free soil-gases for potentially significant saturated hydrocarbons.[4]

By simplifying the sample collection, this modification in the original procedure resulted in a noticeable increase in operating effectiveness. Still more important, however, was the increase in technical effectiveness which resulted thereby. Data of a satisfactory order of accuracy and repeatability could be secured thereby not only from permeable soils open to the atmosphere, but also from below the water table. Much greater uniformity in sampling thus was achieved by analyzing the soil itself instead of the free gas in the soil. As a result, the data of microscopic geochemical prospecting became lower in cost, more readily secured, and much higher in quality.

These increased operating and technical powers appear to have been prerequisites for the major increase in resolving effectiveness which followed, almost at once, the recognition that greater leakage of hydrocarbons to the near-surface soil took place over the edges than over the center of the petroleum accumulation. This recognition was an empirical observation made by Horvitz entirely upon his experimental data alone, and, far from having any theoretical background, actually was propounded counter to such theoretical considerations as had been promulgated to that time.

Primarily as a result of these increased operating, technical and re-

[4] E. E. Rosaire and Leo Horvitz. U. S. Patent No. 2,192,525, issued March 5, 1940.

solving powers, other phenomena of great potential importance were brought to light. Among these were sedimentary hydrocarbon depth-patterns making possible, deep, as well as shallow geochemical well logs and secondary inorganic anomalies, an important group of phenomena closely related to the leakage of significant hydrocarbons.

With the recognition that the near-surface soil hydrocarbon concentrations were found primarily at the edges of the parent accumulation, the question then arose as to whether this was a phenomenon limited to sedimentary hydrocarbons, and which was not present in the earlier investigations of free-soil gas. A review of the earlier work of record, however, showed that this same phenomenon was present in the free soil-gas investigations, but had gone unnoticed by the interested technicians.[5]

Subsequent improvements in analytical procedure increased still further the technical effectiveness, while in 1938, a major improvement in extraction efficiency led to a material increase in finding effectiveness. This last innovation had the net effect of increasing the measured magnitude of anomalous hydrocarbon concentrations without raising their measured normal concentrations. With steadily increasing amounts of data on record, the subsequent developments led to the usual improvement in resolving effectiveness. Finally, a 1940 development was the introduction of a semi-mobile field laboratory whereby the operating effectiveness was increased considerably with no decrease in technical effectiveness.

In this brief review of the histories of the free soil-gas and the sedimentary hydrocarbon prospecting methods, we see a striking example of how the overall prospecting effectiveness remained negligible until a major improvement was made in the resolving power—one which required the complete disregard of previous theoretical postulates based entirely upon considerations of depth and local lateral homogeneity.

CONCLUSIONS

In 1941, then, we are confronted by the following situation. Of the indirect, i.e., structural methods used successfully to date, only one now has any appreciable finding effectiveness. However, the overall structural prospecting effectiveness of that method, the drill, is limited by its low operating effectiveness—the high cost of the data in dollars

[5] E. E. Rosaire. "Discussion and Communications," GEOPHYSICS, Vol. 4, No. 4, (October, 1939), pp. 300–306.

as well as in hours. The other structural methods with a background of success display negligible or nearly negligible structural finding effectiveness in the major petroleum provinces of the U.S.A. And yet, in those same thoroughly explored areas, those same methods have found a multitude of anomalies, of magnitudes frequently greater than those of the associated observational errors, anomalies which are unusable because they cannot be translated into terms of economic significance.

Such a situation developed in refraction prospecting in 1930, leading to the collapse of that method. Yet, subsequent developments have shown that a surprisingly large percentage of the refraction dry holes were close misses rather than outright failures. That demonstration of the empirical significance of non-structural refraction anomalies indicates that the refraction collapse of 1930 could have been avoided by an improvement in the resolving effectiveness of the method.

By comparing the free soil-gas and the sedimentary hydrocarbon prospecting methods, we see the case of a potentially direct approach which displayed negligible prospecting effectiveness until a major improvement in resolving effectiveness took place.

There, and as has been, belatedly, the case in refraction prospecting, the improvement in resolving effectiveness represents a complete departure from the earlier approach of depth predictions based upon the assumption of local lateral homogeneity, and directs attention to lateral variations in near surface sediments which have been found to be significant even in the absence of structural deformation.

Similar increases in the overall prospecting effectiveness of gravity, reflection, and surface geological methods, might well follow from improvements in the resolving power of these methods, for all three of them are capable today of finding anomalies greater in magnitude than the associated observational errors. However, since these demonstratedly findable anomalies are not susceptible of obvious translation into structure, improvements in interpretation would very probably involve discarding the approach based solely upon depth determinations postulating local lateral homogeneity, as was necessary in effecting the increased resolving powers for the refraction and sedimentary hydrocarbon prospecting methods.

There may be a tendency to discount the potential economic importance of an improvement in interpretation methods *per se*, but on the other hand, in 1926, a major renaissance in gravity prospecting was achieved as a result of a bettered resolving effectiveness, which

took the form only of a change in the sign of the objective.

On a purely historical basis, it follows that important advances in prospecting for petroleum will result not only from the introduction of older methods into provinces as yet unexplored by those methods, but also from the introduction of new methods into areas previously explored by older methods. However, the writer predicts with confidence that the major advance in petroleum prospecting will follow a change in objectives, from the indirect objective of geologic structure to the direct objective of the geological, geophysical, and geochemical manifestations of petroleum accumulation itself. As a result, not only will their increased resolving powers effect a major renaissance in the older geological and geophysical methods, but also there will be revealed the fundamental importance of the geochemical methods which first made evident the dynamic nature of petroleum accumulation.

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CONSIDERATIONS ON THE VERTICAL MIGRATION OF GASES*

ROBERT G. NISLE†

ABSTRACT

The vertical migration of a perfect gas saturated with aqueous vapor is studied with a view toward determining what effect it may have on the porous strata through which it passes.

It is found that evaporation of water and the consequent concentration of minerals may occur at certain depths. Further, it is pointed out that the cooling due to the evaporation of water and the expanding gas may be a contributing factor in the appearance of geothermal anomalies.

Suggestions for further study are included.

The method of exploration for deposits of oil and gas by means of an analysis of samples of soil or "soil air" for its hydrocarbon content has received sufficient recognition as an exploration technique to warrant some theoretical consideration. Most of the material in print to date has dealt with the existence of hydrocarbons in the soil, and the areal distribution patterns of these hydrocarbons. It would seem time, then, to study the influence of the migrating gases responsible for these soil hydrocarbons on the strata which they traverse. This paper will deal, therefore, with the migration of gases from a theoretical point of view and determine, if possible, what might be expected in the way of alteration of the sedimentary overburden through which these gases migrate.

ASSUMPTIONS

1. The escape of the gas from the reservoir is assumed to be at a slow uniform rate toward the surface of the earth. This rate to be slow enough that the gas is at all times in thermal equilibrium with its surroundings.

2. There is no heat interchange due to variations of velocity because that has been assumed constant.

3. The gas is assumed to expand as it migrates upward according to the perfect gas law, $pv^n = \text{constant}$.

4. The pressure gradient is assumed to be 0.5 lbs. per square inch per foot of depth, which is approximately that of a normal hydrostatic head.

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5. The temperature gradient is taken as the average of fifteen wells scattered over Texas. This gradient is 0.021°F. per foot of depth.

6. The gas is to be saturated with aqueous vapor at the time of its escape from the reservoir.

DISCUSSION

In view of the above assumptions several points are to be noted:

1. The gas and vapor mixture expands according to a known law and the coordinates of state, p, v, and t, may be calculated at various depths.

TABLE 1

1 *Depth*	2 *t°F.*	3 *T°A.*	4 *p in lbs./in.²*	5 *V cu. ft.*	6 *H_2O Vapor lbs./ft.³*	7 *Total Weight Vapor— lbs.*	8 *lbs. H_2O Vapor Cond.*	9 *lb. H_2O Vapor Evap.*
5000′	172.°	631	2514	1.00	.0168	.0168		
							.0046	
4000′	150.°	609	2014	1.19	.0103	.0122		
							.0027	
3000′	129.°	588	1514	1.54	.0062	.0095		
2000′	108.°	567	1014	2.22	.0035	.0079	.0016	
								.0001
1000′	86.°	546	514	4.23	.00189	.0080		
								.0008
800′	82.8	541	414	5.20	.0017	.0088		
								.0014
600′	78.6	537	314	6.79	.0015	.0102		
								.0017
400′	72.4	533	214	9.92	.0012	.0119		
								.0084
200′	70.2	529	114	18.44	.0011	.0203		
								.0147
100′	68.1	527	64	32.69	.00107	.0350		
								.0206
50′	67.0	526	39	53.45	.00104	.0556		
								.0845
1′	66.0	525	15	140.11	.00100	.1401		

In the above table are given the results of calculations outlined in the text. Depth is in feet; t is temperature in degrees Fahrenheit; T is temperature in degrees Absolute in the English system; p is pressure in pounds per square inch; V is volume in cubic feet; Column 6 gives the density of the aqueous vapor for saturation; Column 7 gives the total weight in pounds of the aqueous vapor in the expanded volume. The last two columns give the weights of vapor condensed or evaporated in the interval shown. The data in Column 6 was obtained from a set of steam tables.

2. The expansion through a porous, permeable medium as outlined above suggests a throttling process in which the thermal poten-

tial, i, remains constant. Now, for a perfect gas the line of constant thermal potential is a constant temperature line. But we have assumed the gas to take the temperature of its surroundings, therefore, the gas can not be said to undergo an isothermal change.

3. No account has been taken of the frictional losses, because the velocity has been assumed slow enough that they may be neglected.

The gas is assumed to start its migration from a depth of 5,000 feet. Now, by Dalton's law for gaseous mixtures we have $p=p'+p''$. In our case p'' is the partial pressure due to the aqueous vapor. It is insignificant compared to the total pressure, however, and we have neglected it in our calculations. In Table I we have the results of calculations made on the basis of our assumptions.

In this table we have:

Column 1. The depth in feet.

Column 2. The temperature in degrees Fahrenheit at that depth calculated on the basis of the temperature gradient assumed.

Column 3. The temperature in degrees Absolute for the English system of units.

Column 4. The pressure at the depth indicated in pounds per square inch.

Column 5. The volume of the gas calculated by the formula $p_1v_1/T_1=p_2v_2/T_2$.

The volume, v, was taken as 1 cu. ft. at 5000 feet, and all calculations made on this basis.

Column 6. The density in pounds per cubic foot of aqueous vapor at saturation of the expanded gas. These values were obtained from a set of Steam Tables.

Column 7. The total weight of aqueous vapor in the expanded volume of gas. (Column 5 times Column 6.)

Column 8. The weight in pounds of the aqueous vapor condensed in the interval indicated.

Column 9. The weight in pounds of the aqueous vapor evaporated in the interval indicated.

An examination of the Table brings to light the fact that from 5000′ to 1000′ (approx.) the total weight of aqueous vapor in the expanded volume decreases continuously. From 1000′ to the surface, however, the total weight increases, indicating a withdrawal of moisture from the strata through which the gas passes. This is indicated in Columns 8 and 9, Table I. In Fig. I, Column 7 in Table I is plotted against the depth.

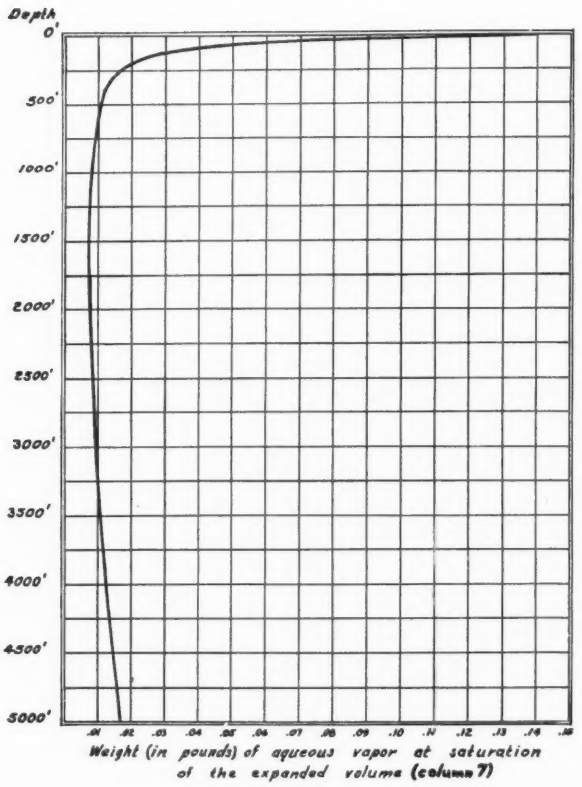

FIG. 1.

ROLE OF THE GAS

Rosaire[1] has suggested that migrating gas might be capable of evaporating moisture from porous media through which it migrates. No attempt was made, however, to put the hypothesis on a scientific basis. In order to study the matter further let us consider the heat changes arising from the expansion of the gas.

From our assumptions we know the expansion is neither isothermal nor adiabatic, but probably somewhere between. Therefore, we may calculate 'n' in the equation $pv^n = \text{constant}$ by means of well known laws of Thermodynamics.

$$n = \frac{ln\ p_2 - ln\ p_1}{ln\ v_1 - ln\ v_2} = 1.04.$$

[1] Rosaire, Dr. E. E.; The Handbook of Geochemical Prospecting, pp. 93, 99: 1939.

Further,

$$ {}_1W_2 : U_2 - U_1 : JQ = k - 1 : 1 - n : k - n, $$

where

${}_1W_2$ = External mechanical work of expansion from state 1 to state 2.

$U_2 - U_1$ = Change of internal heat energy from state 1 to state 2.

JQ = Energy added to, or subtracted from the surroundings.

k = The ratio of the specific heats, taken equal to 1.40.

Therefore, we have

$$ {}_1W_2 : U_2 - U_1 : JQ = 10 : -1 : 9. $$

That is, only one-tenth of the work necessary to produce the expansion is obtained from the gas, the remaining nine-tenths comes from the surroundings. It is evident, therefore, that the heat necessary to evaporate moisture from the soil can not come from the gas. The only other place for it to come from is the surroundings. We must conclude, therefore, that the gas does not directly evaporate water by supplying heat from its internal energy. It merely acts indirectly, or one might say as a carrier for the water vapor.

CONCLUSIONS

In general we may say that a gas migrating under the conditions outlined in this paper may induce evaporation of ground waters, and the consequent concentration of minerals at certain depths. The actual thermal energy required to produce the evaporation, however, comes from the surroundings. Further, the energy represented by the mechanical work of expansion is largely derived from the surroundings. These two facts lead one to suspect that this migrating gas may be a contributing factor in the appearance of geothermal anomalies over structure. For instance, the more rapid escape of gas around the edges of structure results in a more rapid removal of thermal energy, both because of the more rapid evaporation of ground waters, and because of the removal of thermal energy by the expanding gas. And again, the expansion of the gas and the evaporation of the water indicate a cooling due to the removal of thermal energy which is greater at the surface than at depth. These effects would result in a larger thermal gradient (more degrees per foot of depth) near the surface than at depth.

SUGGESTIONS FOR FURTHER STUDY

The discussion contained in this paper is merely intended to cover the general case and it must be understood that each individual case must be dealt with on its own merits. So many factors enter into the problem which will alter the temperature and pressure gradients that in a discussion of this sort an average case must be assumed in order that general conclusions may be arrived at. Therefore, a few suggestions for further study are included.

1. Studies on temperature gradients at various depths to determine to what degree, if any, such gradients might be effected by this migration of gas.

2. Studies to determine actual rates of leakage of gas in various areas of known petroleum deposits.

3. Correlation of information gained in the above studies with hydrocarbon surface patterns and with mineralization found in strata traversed by the drill.

MEMORIAL

HENRY HART PRATLEY

1893–1941

The Society of Exploration Geophysicists and the Scientific World mourn the passing of Henry Hart Pratley at Glendale, California, July 23, 1941. His death occurred following a short illness after a major abdominal operation.

Mr. Pratley was born in Chicago, Illinois, forty-eight years ago. He graduated from the Colorado School of Mines in 1922 after his education had been interrupted by service in the United States Navy during the World War. After graduation he followed the mining game, first in Butte, Montana and later as a very successful mine operator in the Park City-Bingham Canyon districts of Utah.

He became interested in the new field of Seismic prospecting and Geophysics and in 1928 returned to his alma mater to take additional educational work in this branch of science. After several years of magnatometer work in the Midcontinent and Texas oil fields as an independent operator, Mr. Pratley went to California in 1930 to do geophysical work for the Ohio Oil Company. He was one of the first to see the possibilities of applying this type of prospecting to the California oil fields.

In 1936 he became associated with Herbert Hoover, Jr., as Vice-President of the United Geophysical and Engineering Company of Pasadena. He remained in this position until November 1940.

At this time he decided to study the application of seismic prospecting to underground mine work. As a pioneer in this field, he was greatly encouraged by phenomenal success in several "test runs" in Utah mines.

Mr. Pratley is survived by a brother Fred and four sons, Henry Hart Pratley, Jr., Fred Cooper Pratley, Brent McGee Pratley and Bruce McGregor Pratley. He was a member of Beta Theta Pi and Theta Tau fraternities and of El Kalah Shrine Temple, Salt Lake City. He was active in the Society of Exploration Geophysicists, The American Association of Petroleum Geologists and the American Institute of Mining and Metallurgical Engineers.

PATENTS

ELECTRICAL PROSPECTING

U.S. No. 2,249,328, E. E. Rosaire, Iss. 7/15/41, Appl. 8/25/38.

Calibration Means—An electrical network is provided with which to calibrate the receiving circuit of an electrical surveying set-up of the impulse or transient type particularly.

U.S. No. 2,250,024, J. J. Jakosky, Iss. 7/22/41, Appl. 12/1/37.

Method of Determining Underground Structures.—A method of electrical exploration in which the potential at the potential electrodes is maintained constant or permitted to change in a predetermined manner by adjusting the current at the current electrodes as the electrode spacing is varied and measuring the input current. Operational advantages result because of decreased time of application of current with reduction of electrode polarization irregularities.

U.S. No. 2,251,537, G. E. White, Iss. 8/5/41, Appl. 12/16/37. Assign. Standard Oil Development Co.

Electrical Transient Prospecting—A method of prospecting utilizing electrical transients in which the transient from the earth is matched by a simultaneous transient from an electrical network having adjustable constants. The generator providing the earth transient and that supplying the auxiliary network are syncronized at the source.

U.S. No. 2,251,549, P. W. Klipsch, Iss. 8/5/41, Appl. 8/25/38. Assign. E. E. Rosaire.

Mixing Circuit for Electrical Prospecting—A method of prospecting utilizing electrical transients in which the transient from the earth is matched by a simultaneous transient from an adjustable electrical network, said auxiliary transient generator being syncronized with the earth transient by means of the potential picked up from the earth electrodes.

GRAVIMETRIC PROSPECTING

U.S. No. 2,253,472, T. B. Pepper, Iss. 8/19/41, Appl. 7/17/39, Assign. Gulf Research & Development Co.

Apparatus for Submarine Geophysical Prospecting—A submersible container is adapted to rest stationary on the floor of a body of water and to support within it a geophysical instrument such as a gravimeter. Remote control from the surface is provided so that the instrument can be leveled, the moving parts unclamped or clamped, and necessary records made. Photographic recording is employed for the gravimeter.

MAGNETIC PROSPECTING

U.S. No. 2,246,259, L. Machts, Marburg, Germany, Iss. 6/17/41, Appl. 2/17/39.

Apparatus for Making Geophysical or Other Measurements—In a cathode ray tube the deflection of the electron beam is used to measure the variations of a non-axial magnetic field (such as the earth's field). The tube is adapted for operation with a low velocity beam and employs an auxiliary annular electrode in place of the usual screen whereby a deflection of the cathode beam causes it to impinge on the annular electrode.

A measurement of the resulting current provides a measure of the deflection and hence the magnetic field causing the deflection.

U.S. No. 2,252,059, G. Barth, Iss. 8/12/41, Appl. 12/24/37, Assign. Siemens Apparate und Maschinen Gesellschaft m.b.H., Berlin, Germany.

Method and a Device for Determining the Magnitudes of Magnetic Fields—In effect the device is equivalent to a saturated core reactor such that variations in the magnetic bias of an elongated core due to an ambient magnetic field causes a change in the impedance of an a.c. winding. The change in a.c. current in this circuit is then a measure of the ambient field. Feed-back amplification is disclosed whereby a rectified component of the a.c. excitation is applied to an auxiliary d.c. winding and in addition an adjustable d.c. may be applied to attain the proper operating point on the magnetization curve. The core is highly permeable material such as permalloy.

SEISMOGRAPH PROSPECTING

U.S. No. 2,253,358, P. S. Williams, Iss. 8/19/41, Appl. 8/13/38, Assign. Standard Oil Development Co.

Seismic Exploration Method—Receiving circuit for reflected seismic waves provides for algebraically combining in pairs the oscillations from a plurality of pickups and converting them to unidirectional impulses. The rectified output of paired channels may be recorded separately or combined into a single "average" trace.

WELL LOGGING

U.S. No. Re. 21,832, F. W. Huber (Deceased), Iss. 6/17/41, Appl. 2/23/39. Assign. Schlumberger Well Surveying Corp.

Method of and Apparatus for Electrically Exploring Earth Formations—A d.c. method using a wheatstone bridge one arm of which comprises the resistance between a single electrode traversing the well bore and a grounded terminal at the surface. The bridge is adjusted to nominal balance and resistance variations encountered in the traverse are indicated or recorded by the galvanometer deflection.

U.S. No. 2,245,700, W. D. Mounce, Iss. 6/17/41, Appl. 10/8/37, Assign. Standard Oil Development Co.

Resistance Thermometer—For measuring temperature variations in a well bore a resistance element with a resistance low compared to that of the well fluid is provided whereby the thermometer element may be in intimate contact with the well fluids and thus minimize the thermal lags associated with insulated elements.

U.S. No. 2,246,319, R. Ring, Iss. 6/17/41, Appl. 1/13/40, Assign. Sperry-Sun Well Surveying Co.

Well Surveying Instrument—Inclination of the well bore is indicated by mark produced electrolytically on a record by a stylus attached to a pendulum.

U.S. No. 2,246,460, C. B. Bazzoni and J. M. Pearson, Iss. 6/17/41, Appl. 8/2/38, Assign. Sperry-Sun Well Surveying Co.

Electrical Prospecting Apparatus—A design disclosing concentric leads, through a

high pressure seal for connecting electrical coring equipment with cables from the surface.

U.S. No. 2,246,519, P. Jones, Iss. 6/24/41, Appl. 3/18/39.

Drift Recorder—For measuring the deviation of a well bore, a beam of light through a central perforation in a light sensitive record disk is reflected from a spherical mirror floated on mercury and focused on the sensitized under-surface of the record disk. The floating mirror serves as a level; hence the deviation of the recorded spot on the record indicates the deviation of the bore hole.

U.S. No. 2,247,417, D. Silverman and R. W. Stuart, Iss. 7/1/41, Appl. 3/6/40, Assign. Stanolind Oil and Gas Company.

Electrical Logging—One or more insulated electrodes are incorporated in the lower end of a drill stem. Without removing the drill stem a cable may be lowered through the drill pipe and the electrodes contacted by special contacting devices attached to the end of the cable.

U.S. No. 2,248,101, R. W. Lohman, Iss. 7/8/41, Appl. 6/13/38, Assign. Schlumberger Well Surveying Corp.

Method and Apparatus for Investigating Subterranean Strata—A method of well surveying comprising investigation of the magnetic field intensity (flux density) along a well bore traverse. Apparatus disclosed comprises a rotating coil generator or alternatively a galvanometer-type coil operating in the ambient magnetic field with photoelectric deflection indicator.

U.S. No. 2,248,982, J. R. Gillbergh, Iss. 7/15/41, Appl. 1/4/39.

Method and Apparatus for Determining the Character and Points of Ingress of Well Fluids—Apparatus for conditioning a well and then testing electrically the character of the well fluid at a plurality of points simultaneously by spaced electrodes.

U.S. No. 2,249,108, R. F. Beers, Iss. 7/15/41, Appl. 4/27/39.

Means for Analyzing and Determining Geologic Strata—The velocity of sound waves in the various strata is measured by lowering two seismometers spaced relatively close to each other and receiving waves generated at the surface by a continuously operating oscillator.

U.S. No. 2,249,769, E. G. Leonardon, Iss. 7/22/41, Appl. 11/28/38, Assign. Schlumberger Well Surveying Corp.

Electrical System for Exploring Drill Holes—One or two insulated electrodes in the drill bit are connected by conductors to the surface. Spontaneous potentials, resistivity of the formation around the drill bit, and resistivity of material between bit and surface are measured as drilling progresses.

U.S. No. 2,250,703, W. J. Crites and W. C. Rodgers, Iss. 7/29/41, Appl. 11/18/38, Assign. Phillips Petroleum Co.

Apparatus for Locating Casing Seats—A change in the magnetic permeability in the

vicinity of an open-cored coil lowered in a well is detected by use of an alternating current bridge circuit.

U.S. No. 2,251,817, L. F. Athy and H. R. Prescott, Iss. 8/5/41, Appl. 4/18/38, Assign. Continental Oil Co.

Method of Borehole Logging—Acoustical logging in which an oscillator is lowered in a well and excited by an alternating e.m.f. The acoustic impedance of the adjacent strata is reflected in the measured motional electrical impedance of the oscillator. Variable frequencies may be used and strata are identified and correlated by noting the frequencies of maximum and minimum energy transmitted by the strata.

U.S. No. 2,251,900, B. S. Smith, Iss. 8/5/41, Appl. 9/24/38, Assign. Fifty per cent J. J. Kane.

Well Surveying—Presumably making use of the "Hall effect," the variations are observed in an otherwise steady current between an electrode traversing a well bore and adjacent formations when the electrode-earth system is simultaneously subjected to an applied magnetic field.

U.S. No. 2,253,485, C. B. Bazzoni and J. Razek, Iss. 8/19/41, Appl. 8/2/38, Assign. Sperry-Sun Well Surveying Co.

Electrical Prospecting Method and Apparatus—Improvements over U.S. Pat. 2,167,630 involving a high frequency electromagnetic energy source and a suitable detector and recorder of such radiation, means for lowering said apparatus in a well bore whereby variations in the electromagnetic field resulting from different materials traversed by the exploring unit may be recorded. Monitoring means may be arranged for temporary association with the apparatus within the exploring element for checking its operation before entrance into the well bore. Means for correlating records with depth include transmission of acoustic waves down the supporting cable.

CONTRIBUTORS

CHARLES HEWITT DIX

CHARLES HEWITT DIX received his B.S. degree in Physics from the California Institute of Technology in 1927, and his Ph.D. in Mathematics from the Rice Institute in 1931. He was an instructor in Mathematics at Rice from 1929 until 1934, when he became associated with the Humble Oil and Refining Company as a research geophysicist. In 1939 he became geophysicist for the Socony Vacuum Oil Company, handling the research aspects of foreign geophysical exploration. He is the author of nine papers on geophysical and mathematical subjects in addition to unpublished research reports. Dr. Dix's work has covered all fields of petroleum geophysics, but the principal emphasis has been on new field methods and new methods of interpretation of field data. A natural side-line of a thorough study of field methods and interpretive technique is research in instrument design, which Dr. Dix has followed for a number of years. He is a member of the American Mathematical Society, the American Physical Society, the Seismological Society of America, the American Geophysical Union, and the Society of Exploration Geophysicists.

JOSEPH D. EISLER received his B.S. degree from Massachusetts Institute of Technology in 1932. From 1932 to 1935 he was engaged in heat research at the Institute and from 1935 to 1936 was employed as Research Assistant by the Geophysical Department of M. I. T. In 1936 he joined the research staff of the joint Laboratories of the Western Geophysical Co. and the Stanolind Oil and Gas Co. by whom he is still employed. He is engaged in instrumental research in seismic and other fields of geophysics. He is a member of the Society of Exploration Geophysicists.

JOSEPH D. EISLER

NORMAN A. HASKELL received his B.S. degree in 1927 and his Ph.D. in 1936, both from Harvard. He has been engaged in geophysical work since 1936 when he accepted a position with the Rieber Laboratory. He became associated with the Western Geophysical Company the following year. His recent work work has been concerned with seismic velocities and other problems relating to the interpretation of the results of seismic surveys. In August 1941 he joined the scientific staff of the Columbia University National Defense Laboratories. He is a member of the American Physical Society, the Seismological Society of America, the American Association of Petroleum Geologists, and the Society of Exploration Geophysicists.

NORMAN A. HASKELL

FREDERICK ROMBERG

FREDERICK ROMBERG received his B.A. degree from the University of Texas in 1929, and his A.M. in Physics from Harvard in 1932. He spent the year 1929–30 in the geophysical department of the Humble Oil and Refining Company, and 1932–33 as a tutor in the Physics Department at Harvard. In 1933 he was employed by Geophysical Service, Inc. He was for some years chief of a seismograph field party, and later worked on the development and application of new interpreting methods, especially with respect to horizontal changes in seismic velocities. At present he is on leave of absence from Geophysical Service, Inc. and is engaged in maintenance and development of gravity meters in the LaCoste-Romberg laboratories. He is a member of the Seismological Society of America and of the Society of Exploration Geophysicists.

ESME EUGENE ROSAIRE

ESME EUGENE ROSAIRE received his B.S. degree in 1920, his M.S. degree in 1921, and his Ph.D. degree in 1926, all from the University of Chicago. He has been engaged in geophysical prospecting since 1925, when he was the first man in the field for the Geophysical Research Corporation. In 1932, he assisted in the organition of the Independent Exploration Company, serving as president and general manager of that company until 1937. He resigned in that year to organize Subterrex, of which he is owner and general manager. His experience has covered refraction, reflection, geoelectrical and geochemical prospecting for petroleum. In 1926 he was in charge of the Gulf G.R.C. crew which discovered the Moss Bluff salt dome, Liberty County, Texas, the first such discovery by an American refraction crew. At present his activities are directed at the development and introduction of geochemical methods of prospecting for petroleum.

Dr. Rosaire is a charter member of the Society of Exploration Geophysicists, of which he was Vice President in 1930, and President in 1934. He is a member of the American Association of Petroleum Geologists, the American Institute of Mining and Metallurgical Engineers, the Society of American Military Engineers, and a fellow of the American Association for the Advancement of Science and the Institute of Petroleum Technologists.

WILLIAM E. STEELE, JR., received his B.S. degree from Texas A. and M. College in 1933. He has been in geophysical work since 1934 when he joined the Independent Exploration Company. Since April, 1941, he has been in the Signal Corps of the U. S. Army, where he is at present. Mr. Steele's work has been primarily in interpretation of reflection seismograph data. He is a member of the Society of Exploration Geophysicists and the Houston Geological Society.

WILLIAM E. STEELE, JR.

EDGAR J. STULKEN received his B.A. degree from the University of Texas in 1934 and in 1935 was awarded his M.A. degree by the same school. In 1935 he became associated with Geophysical Service, Inc., Dallas, Texas, and remained with that organization until 1940. In September, 1940, he joined the staff of Allen Military Academy, Bryan, Texas, as Instructor of Mathematics where he is still located. Mr. Stulken is a member of the Society of Exploration Geophysicists.

EDGAR J. STULKEN

THE SOCIETY ROUND TABLE

ANNUAL MEETING, DENVER, COLORADO, WEEK OF APRIL 20, 1942

The Annual meeting of the Society will be held in Denver, Colorado, during the week of April 20, 1942. The A.A.P.G. will be convening during the same week and will have their headquarters at the Cosmopolitan Hotel. At the time of going to press our own arrangements are not completed but we confidently anticipate being able to share this headquarters hotel with the A.A.P.G.

The Program and Arrangements Committee of the Society, of which the Vice President is Chairman, has not yet been appointed; members of this committee will however, in due course approach prospective authors for papers to be presented at this meeting. If any member has a paper which he desires to present, but is not approached by a member of the committee, he should submit this paper directly to the Chairman, Dr. F. Goldstone, P.O. Box 2099, Houston, Texas.

MEMBERSHIP APPLICATIONS APPROVED FOR PUBLICATION

The Executive Committee has approved for publication the names of the following candidates for membership in the Society. This publication does not constitute an election, but places the names before the membership at large. If any member has information bearing on the qualifications of these nominees, he should send it to the Secretary within thirty days. (Names of sponsors are placed beneath the name of each nominee.)

ACTIVE

Harold Clayton Bickel
Malcolm Riess, J. D. Berwick, A. B. Bryan

Herbert Wray Hole
J. A. Moore (Article III-D-1 of the Constitution)

George Ross Johnson
J. D. Berwick, George E. Wagoner, R. W. Gemmer

Albert Klein-Haneveld
F. A. Van Melle, Alfred J. Herzenberg (Article III-D-1 of the Constitution)

Rodolfo Martin
Herbert Hoover, Jr., Cecil E. Reel, Hugh C. Shaeffer

Chalmer John Roy
Paul Weaver, John Doering, Charles Gill Morgan

Rudolph Frederick Weichert, Jr.
D. C. Skeels, Bela Hubbard, Robert J. Watson

Gordon M. Wenz
E. V. McCollum, Neal J. Smith, J. G. Ferguson

ASSOCIATE

Ralph Carroll Holmer
L. M. Mott-Smith, C. A. Heiland, Dart Wantland

Vincent James Michael Mercier
Charles C. Williams, John A. Gillin, J. H. Pernell

Alex Semryck
William Schriever, W. O. Bazhaw, L. I. Freeman

STUDENT

Allen Minard Rugg, Jr.
J. E. Hawkins, Dart Wantland, C. A. Heiland

TRANSFER FROM ASSOCIATE TO ACTIVE

R. S. Epperson
J. F. Gallie
M. J. Pellillo

ADDITIONAL APPLICATIONS APPROVED FOR PUBLICATION

ACTIVE

Joseph Horace Deming
L. A. Scholl, Jr., Roy L. Lay, George D. Mitchell, Jr.
Edwin M. Swift
C. A. Heiland, Dart Wantland, W. S. Levings
Warwick Mellor Whitley
J. E. Gunn, George B. Lamb, Russell B. Kerbow
Arthur Bachman Wood
F. Goldstone, D. S. Hughes, G. W. Rulfs, Jr.

ASSOCIATE

Ivan Victor Scherb
C. R. Wallace, Roy L. Lay, L. A. Scholl, Jr.
Robert Yule Walker, Jr.
L. A. Scholl, Jr., C. R. Wallace, Roy L. Lay

NOTICE OF CHANGE OF ADDRESS OF BUSINESS MANAGER

Members and subscribers please note:

The business office of the Society of Exploration Geophysicists has been returned to Houston, Texas, from Austin. As heretofore, all correspondence pertinent to editorial matters should be addressed to R. D. Wyckoff, Editor of *Geophysics*, P. O. Box 2038, Pittsburgh, Pennsylvania. All other Society correspondence should be addressed to:

J. F. GALLIE, Business Manager
SOCIETY OF EXPLORATION GEOPHYSICISTS
P. O. Box 2585
Houston, Texas

PERSONAL ITEMS

(Members are requested to forward material for inclusion in PERSONAL ITEMS to J. F. Gallie, P. O. Box 2585, Houston, Texas.)

The business office of the Society of Exploration Geophysicsts has been returned to Houston from Austin, Texas. The new postal address is Box 2585.

The Houston offices of Geophysical Service, Inc. have been moved from 825 to 821 Esperson Building. H. B. Peacock, President of the Society, J. W. Thomas, and V. W. Teufel are located in the new offices.

The Executive Committee of the Society has approved a reduction in the price of single copies of *Geophysics*. A detailed announcement of this reduction and of the available numbers will be found in the advertising section of this issue.

CHESTER J. DONNALLY, Vice President of the Western Geophysical Company, advises that an office has been opened recently at 1438 Mellie Esperson Building, Houston, Texas. Mr. Donnally is supervising all midcontinent work, with his office located in Houston.

G. C. HOWARD, of the Carter Oil Company, is now stationed in Duncan, Oklahoma, with the postal address Box 1071.

ETHEL WARD MCLEMORE has moved from Dallas, Texas, and may be addressed c/o Captain R. H. McLemore, Air Corps Technical School, Fort Logan, Colorado.

HOWARD ITTEN, newly-elected student member and recent graduate of the Colorado School of Mines, is with the Stanolind Oil and Gas Company, P. O. Box 249, Scott City, Kansas.

E. D. WILLIAMS, President of the Universal Exploration Company, advises that offices have been moved from the Shell Building to the new laboratory at 2044 Richmond Road, Houston, Texas.

SAM ZIMERMAN, of the Carter Oil Company, has transferred from Minot, North Dakota, to Box A, Buffalo, South Dakota.

HERBERT J. FERBER, with the Western Gulf Oil Company, is located in Fresno, California, with the postal address P. O. Box 627.

The Independent Petroleum Association of America recently mailed out a mimeographed booklet entitled "The Oil Industry in Your State." A copy is in the Society files in Houston, Texas.

Captain GEORGE B. KAISER has been transferred from Long Branch, New Jersey, to 532 20th Street N. W., Washington, D. C., where he is on duty in the office of the Chief Signal Officer.

JAMES C. TEMPLETON, Managing Director of the Geophysical Prospecting Co., Ltd., advises that offices have been moved to 6 Neville Court, Abbey Road, London, N. W. 8, England.

JOHN M. GOLDEN, formerly with the Independent Exploration Company of Houston, is now with the Elflex Company, 1706 Niels Esperson Building, Houston, Texas.

G. W. POSTMA, with Bataafsche Petroleum Maatschappij, has been transferred to Pangkalan Brandan, S. O. K., D. E. I.

ALEJANDRO DEL RIO M., June graduate of the Colorado School of Mines, is serving with the National Geophysical Company of Dallas prior to returning to Carrera 14 No. 62-16, Bogota, Colombia, S. A., this November.

CHARLES C. LASH has removed from Hanford, California, where he was serving with the Western Geophysical Company, to 6961 Maple Street, Takoma Park, D. C. He is there serving in a confidential capacity in the Naval Ordnance Laboratory.

JAMES J. ROARK, of the Carter Oil Company, requests that, because of frequent moves, correspondence be sent him at Yellville, Arkansas.

HOLLOWAY H. FROST, formerly located with the Mott-Smith Corporation in Houston, is now with the Socony-Vacuum Oil Co., C. A., Apartado 246, Caracas, Venezuela, S. A.

L. I. BROWN has been transferred to the California Company at New Orleans, Louisiana. His mailing address is 1905 South Carrollton Avenue, Apt. E.

MARK MILSTEIN has moved from Melbourne, Australia, to Toronto, Ontario, Canada. He is temporarily with Sladen-Malartic Mines, Ltd., while his permanent address is 155 McPherson Avenue, Toronto.

BILLY E. RICHARDS, newly-elected student member, is serving as assistant computer with the United Geophysical Company, Box 255, Durant, Oklahoma.

HOWARD L. COBB has returned from Hanover, New Hampshire, where he was taking graduate work at Dartmouth, to employment with the Mott-Smith Corporation, 909 Shell Building, Houston, Texas.

Dr. MORTON MOTT-SMITH, formerly with the Independent Exploration Company, is serving as Staff Writer with Science Service, 1719 N Street, N. W., Washington, D. C.

T. F. JOHNSTON, formerly in the E. E. Dept. of Iowa State College, Ames, Iowa, is serving with the National Defense Research Committee, U. S. Navy Radio and Sound Laboratory, Point Loma, San Diego, California.

HARRISON E. STOMMEL, June graduate of the Colorado School of Mines, is serving as Helper on a Gravimeter Party with the Magnolia Petroleum Company, P. O. Box 126, Ada, Oklahoma.

REED L. MCCARTNEY, formerly with the Independent Exploration Company, has been ordered to duty with the 23rd Infantry, Ft. Sam Houston, Texas.

F. H. AGEE, of the National Geophysical Company, has moved from Merryville, Louisiana, to 123 South Greenwood Street, Lebanon, Tennessee.

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SPECIAL NOTICE

Effective November 1, 1941, the price of single issues of the Society Journal, GEOPHYSICS, is reduced to $1 per copy for members of the Society of Exploration Geophysicists and $2 for non-members. This reduction has been made by the Executive Committee in the belief that it will enable interested members and subscribers to complete their files of GEOPHYSICS reasonably and promptly, and that the more widespread distribution of all available geophysical data will thus be in keeping with the avowed object of the Society—the promotion of the science of geophysics, especially as it applies to exploration.

Publications since 1935 are available with the exception of Volume I, Number 1, for January, 1936. The supply of some numbers is limited, however, and it is urged that orders be placed promptly. A cumulative index of articles is available to those who did not receive a copy with the July, 1940 issue of GEOPHYSICS.

Address all inquiries to

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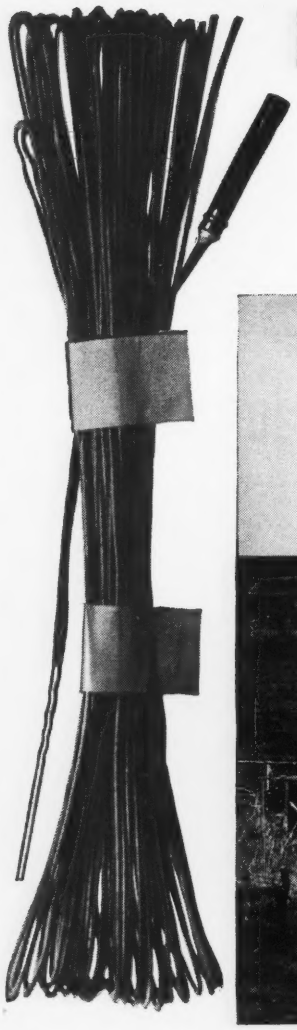

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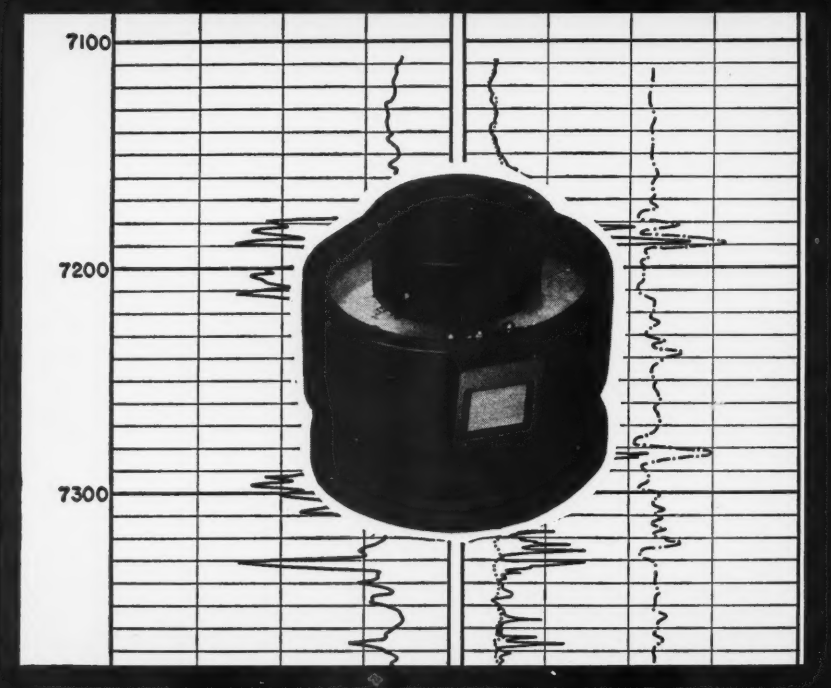

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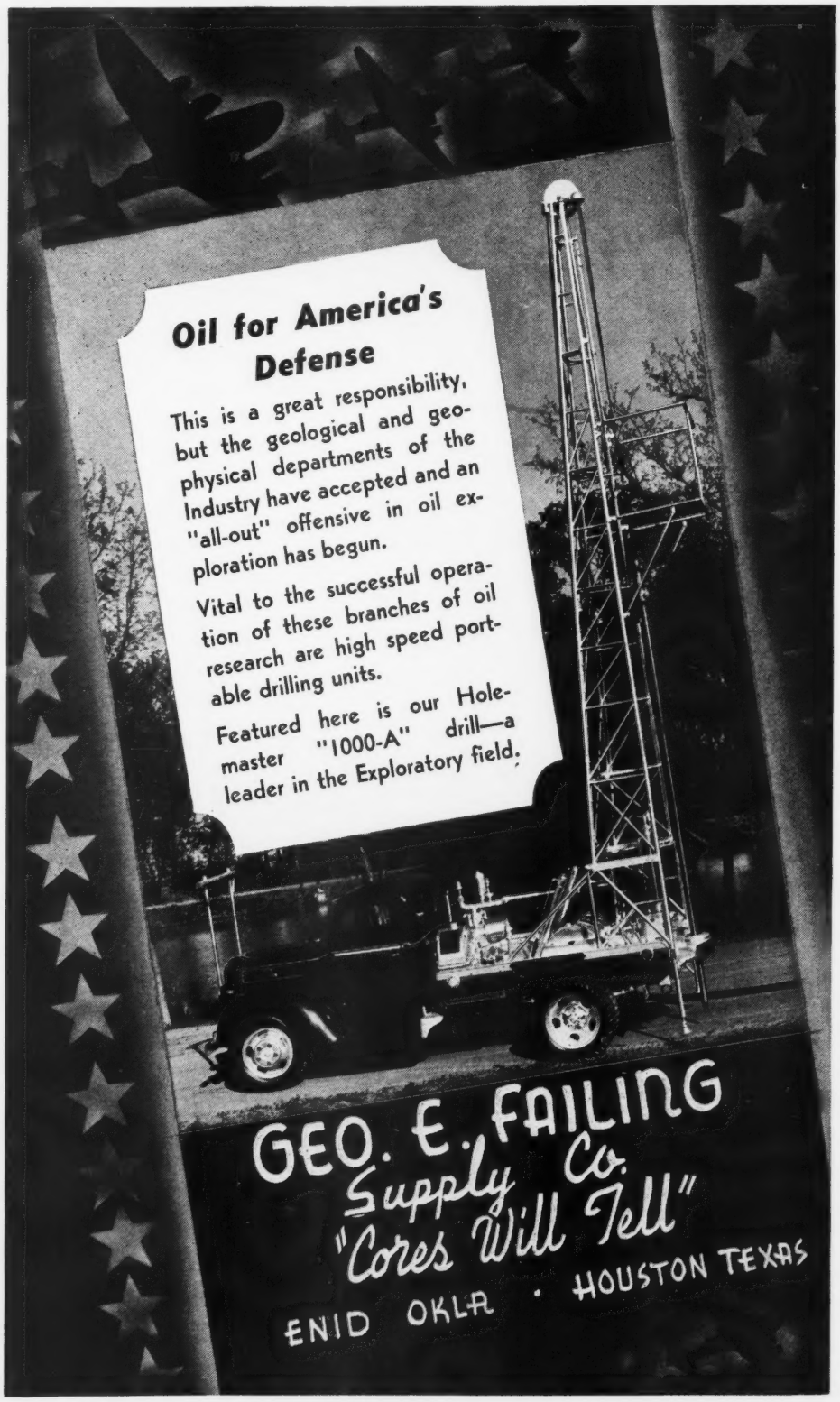
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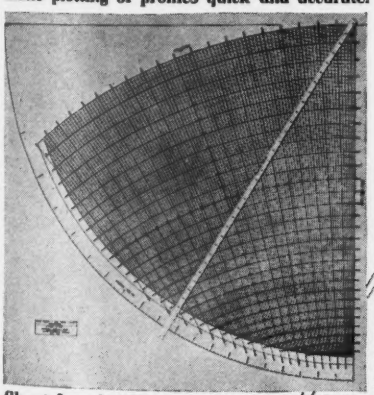

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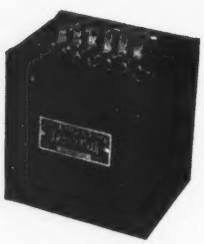

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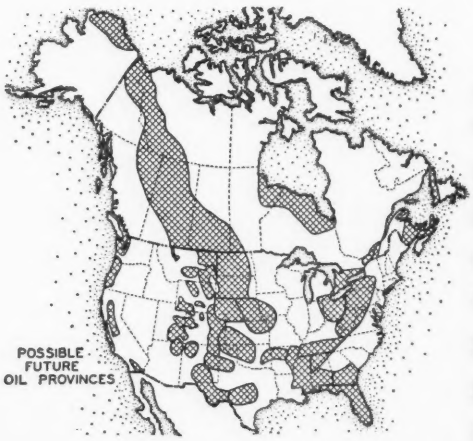

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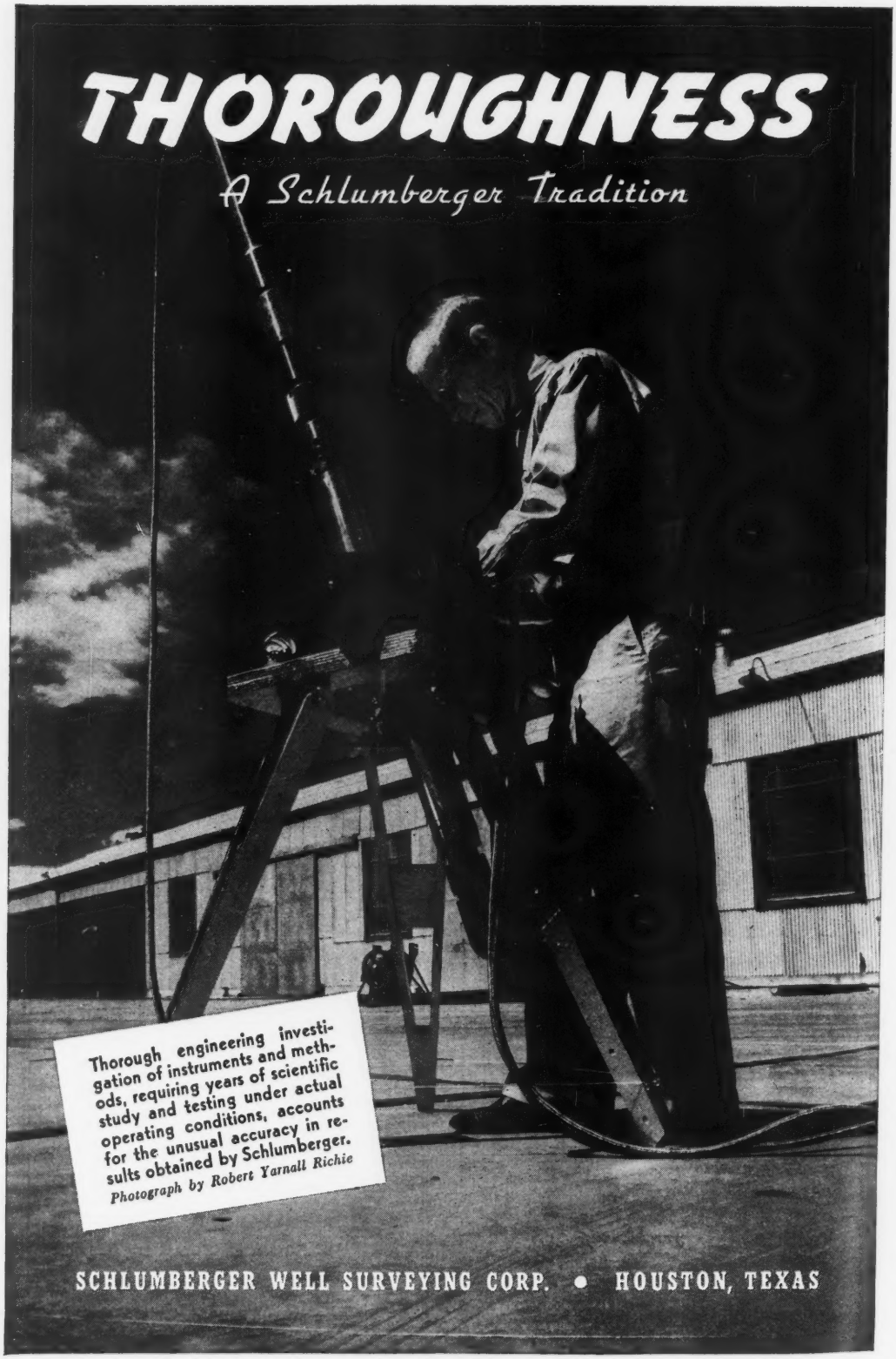
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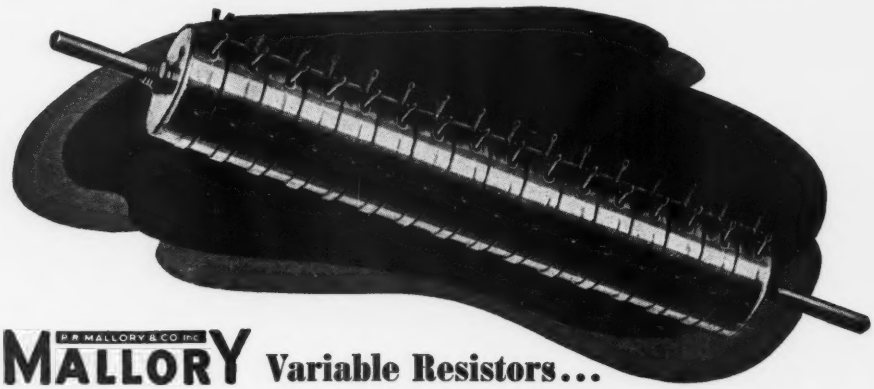

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